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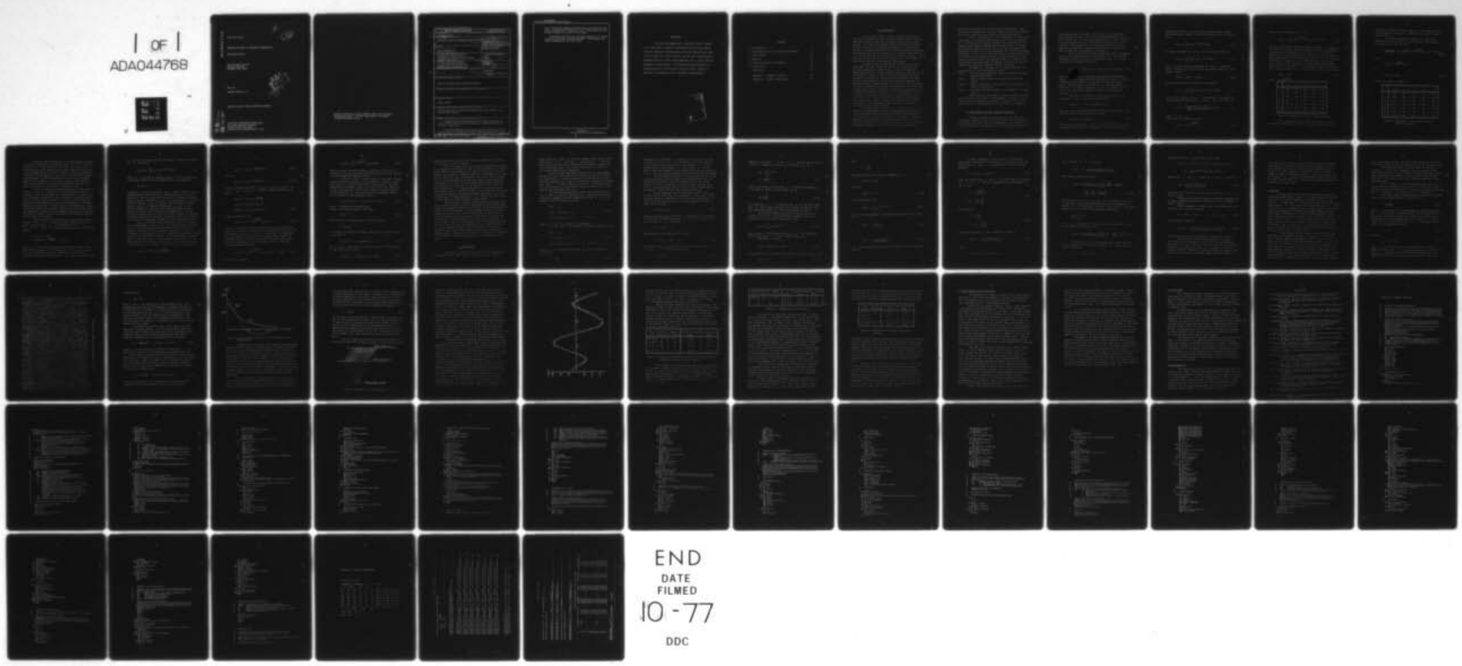

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SIMULATION STUDY OF AIRBORNE GRADIOMETRY

Klaus-Peter Schwarz

The Ohio State University
Research Foundation
Columbus, Ohio 43212

May 1977

Scientific Report No. 8

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A simulated field of gravity anomalies and second-order gradients is used to study the recovery of gravity anomalies at ground level from measurements at flight level.

The spectral properties of gravimetric quantities simulated by an array of mass points are discussed for first and second order gradients and some rules are derived for generating such fields. To study the effect of observational

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) errors, uncorrelated normally distributed errors and correlated errors from a stationary Markov sequence are used. Results agree well with estimates obtained in a previous accuracy study.

Computations show that a fast operational program can be obtained by using a least-squares collocation procedure. The programs and a sample computation are part of the report.

FOREWORD

This report was prepared by Dr. Klaus-Peter Schwarz, assistant to Dr. Helmut Moritz, Professor, Technical University at Graz and Adjunct Professor, Department of Geodetic Science of The Ohio State University, under Air Force Contract No. F19628-76-C-0010, The Ohio State University Research Foundation Project No. 4214A1, Project Supervisor, Urho A. Uotila, Professor, Department of Geodetic Science. The contract covering this research is administered by the Air Force Geophysics Laboratory (AFGL), Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Project Scientist.

CONTENTS

1. Introduction

In a previous report (Schwarz, 1976) the accuracy of airborne gradiometry has been studied and some conclusions have been drawn about optimal point configurations and data combinations. This report supplements some of the previous investigations. Not much can be added with respect to the expected accuracy. Simulation studies display the behaviour of individual experiments only and are therefore not suited to check results of an accuracy study. The interest of a simulation study is therefore not so much in the field of accuracy but in the domain of operational realization and optimal performance.

In order to get an operational program for airborne gradiometry the most important problem to cope with is the efficient handling of large amounts of data. The proposed measuring system will produce about 250 observations per profile and degree. In order to cover a $20^\circ \times 25^\circ$ area with profiles spaced at 1° we have to treat 130 000 measurements. For mean gravity values below $1^\circ \times 1^\circ$ we have to use 20' spacings and the above number of measurements will triple. It has been shown in Schwarz (1976) how the number of observations can be reduced without significantly impairing the accuracy of the results. For an operational program, however, it will be necessary to use all information available. Not so much to increase accuracy but to make results more reliable. Therefore, we have to incorporate both viewpoints in such a program. Efficiency asks for data selection in each computation step. Reliability requires the processing of all data available.

A second reason to carry out a simulation study is the treatment of pathological error situations. Accuracy studies are usually performed under the assumption that the observational errors follow a Gaussian distribution. With complex measuring systems on a moving base this assumption may not be realistic. Besides correlated errors biases are likely to occur in airborne gradiometry. Effects of this kind are easy to simulate and results are important for the planning of experiments. Thus, the actual development of the error budget coming from different sources will help to plan an effective updating procedure.

Finally, the checking of such a program in a controlled experiment is a worthwile exercise by itself. Not only because of its complexity but because instabilities stemming from the downward continuation problem should be controlled in the best way possible. Special care must be taken that the methods for generating the data are truly independent of the data processing procedures. If this can be achieved simulation studies will give a reliable base to handle real data. Large differences in the results from actual and from simulated data will indicate that the mathematical model needs refinement. The nature of the refinement can often been guessed from the simulation.

The numerical treatment of the problem requires the consideration of the following four steps:

Step 1 ... Gravity and gradiometry data are generated at ground and at flight level.

Step 2 ... Data at flight level are corrupted by the error model.

Step 3 ... Gravity anomalies at ground level are estimated from data at flight level.

Step 4 ... Estimated and model anomalies are compared at ground level.

These steps will be covered in the next three sections. Data simulation in section 2, error models in section 3, and estimation and comparison in section 4. Section 5 will give a short review of the programs which are listed in an appendix.

2. Mass Models and Their Spectral Properties.

The basic assumption underlying the simulation of gravimetric quantities by a point mass model can be formulated in the following way: The field generated by such a model in a limited region can be regarded as a sufficient approximation of the anomalous gravity field in this region. In a number of applications the actual field can only be described by statistical parameters. In such a case the above would imply that the statistical properties of the simulated field can also be considered as approximations of the actual quantities. This assumption has important theoretical and practical implications. Some of them will be examined in the sequel.

Let us first consider the simple case that all anomalous masses are concentrated on a plane at depth z_1 and that the simulated function is wanted on a parallel plane z_2. Using Cartesian coordinates we can write

$$g(x,y,z_2) = \iint_{-\infty}^{+\infty} h(x-x',y-y',z_2-z_1)f(x',y',z_1)dx'dy' \qquad (2.1)$$

where f is the density function representing the anomalous masses, h is the upward continuation operator, and g is the simulated gravity function. Since we will use the spectral density function later on we will call f the mass density function in the sequel. Obviously, g will be singular at z_1, i.e. it is only defined above this plane, and we have the conditon $z_2 > z_1$.

Equation (2.1) defines a double convolution of the functions h and f which may be written as

$$g(x,y,z_2) = h(x,y,z_2-z_1) ** f(x,y,z_1) \qquad (2.2)$$

where $*$ is the convolution symbol. Since we consider only integrations in the (x,y)-plane we will write

$$g(x,y) = h(x,y) ** f(x,y) \; , \qquad (2.3)$$

keeping in mind that both g and f refer to a fixed z. Let us assume that the Fourier transforms of all three functions in equation (2.3) exist. We will denote them by capital letters and reserve small letters for the data domain. As an example we have

$$G(u,v) = \iint_{-\infty}^{+\infty} g(x,y)e^{-i(ux+vy)}dxdy \tag{2.4}$$

as Fourier transform of $g(x,y)$ and the inverse relation

$$g(x,y) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} G(u,v)e^{i(ux+vy)}dudv \ . \tag{2.5}$$

$G(u,v)$ is also called the spectrum of $g(x,y)$. Forming all three transforms we can make use of the simple relations in the spectral domain

$$G(u,v) = H(u,v) \cdot F(u,v) \tag{2.6}$$

and using the inversion formula (2.5) we obtain

$$g(x,y) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} H(u,v)F(u,v)e^{i(ux+vy)}dudv \ . \tag{2.7}$$

The transfer function $H(u,v)$ is determined by the geometrical relations between the planes z_1 and z_2 and is of the form

$$H(u,v) = \iint_{-\infty}^{+\infty} \frac{d\cdot e^{-i\{u(x-x')+v(y-y')\}}}{\{(x-x')^2+(y-y')^2+d^2\}^{3/2}} \, dx'dy' \tag{2.8}$$

where $d = z_2 - z_1$.
Evaluating the integral we obtain

$$H(u,v) = 2\pi e^{-d\sqrt{u^2+v^2}} \tag{2.9}$$

and we can write formula (2-7) as

$$g(x,y) = \frac{1}{2\pi} \int\int_{-\infty}^{+\infty} e^{-d\sqrt{u^2+v^2}} F(u,v) e^{i(ux+vy)} du dv \qquad (2.10)$$

Equation (2.10) shows that the transfer function is a smoothing function which affects the high frequencies most. The degree of the smoothing is dependent on the size of d, i.e. on the separation of the two planes. To illustrate this point table 2.1 gives smoothing factors for different values of d and different frquencies. To make the results applicable to the mass model used later on, a grid of 61 by 61 equidistant mass points has been chosen. The variable d is expressed in units of the mass point spacing. Since $H(u,v)$ has circular symmetry we have used

$$H(w) = 2\pi e^{-dw} \qquad (2.11)$$

where $w = \sqrt{u^2+v^2}$.

w	d				
	0.5	1.0	1.5	2.0	2.5
1	.950	.902	.857	.814	.773
5	.773	.597	.462	.357	.276
10	.597	.357	.213	.127	.076
15	.462	.213	.099	.045	.021
20	.357	.127	.045	.016	.006
25	.276	.076	.021	.006	.002
30	.213	.045	.010	.002	.000

Table 2.1 Smoothing of gravity anomaly spectrum

It is apparent from this table that most of the high frequency information is lost if d becomes larger than the mass point spacing. This will result in a very smooth gravity field because its structure is determined by a few low frequencies only.

A similar consideration applies for second-order derivatives. We will show it for the vertical derivative of g . Using equation (2.10) we obtain

$$\frac{\partial g(x,y,z_1)}{\partial z_1} = \frac{1}{2\pi} \int\int_{-\infty}^{+\infty} \sqrt{u^2+v^2}\, e^{-d\sqrt{u^2+v^2}} F(u,v) e^{i(ux+vy)} du dv \ . \tag{2.12}$$

Thus

$$H_z(u,v) = \sqrt{u^2+v^2}\, H(u,v)$$

or

$$H_z(w) = w\, H(w) \ . \tag{2.13}$$

Results are given in table 2.2.

w	h				
	0.5	1.0	1.5	2.0	2.5
1	.950	.902	.857	.814	.773
5	3.865	2.987	2.309	1.785	1.380
10	5.975	3.570	2.133	1.274	.761
15	6.928	3.200	1.478	.682	.315
20	7.140	2.549	.910	.325	.116
25	6.399	1.904	.525	.145	.040
30	6.399	1.365	.291	.062	.013

Tabel 2.2 'Smoothing' of second-order vertical gradient spectrum

Since the high frequencies of the second-order gradients are strongly amplified the smoothing effect of $H(w)$ is counter-balanced if d does not become too large. Thus, even with d twice the mass point spacing we can expect adequate information on the frequencies up to 15 or 20. There is not enough empirical information at the moment on the actual variation of the second-order gradients to decide whether a field with such frequency content is too smooth. It is obvious, however, from table 2.2 that by varying the grid density of the mass points and their depth we can model a wide range of different fields.

So far no assumptions have been made on the mass density function $f(x',y',z_1)$ except that it should possess a Fourier transform. Thus, we can model e.g. a given gravity anomaly field in a completely deterministic way by properly distributing point masses at certain depths. However, if the field to be simulated can only be characterized by statistical parameters a different approach must be taken. In such a case we are looking for a mass distribution which will generate a field with the desired statistical properties. Obviously, the stochastic characteristics of the mass density function and the influence of the transfer function must be taken into account.

Let us consider a two-dimensional wide-sense stationary process $f(x,y)$, i.e. a process which has constant mean value and an autocorrelation r_{ff} which depends only on $\xi = x_1 - x_2$ and $\eta = y_1 - y_2$. Thus, it is characterized by its first and second moments

$$E\{f(x,y)\} = \text{const.}$$
$$E\{f(x+\xi, y+\eta) \cdot \overline{f(x,y)}\} = r_{ff}(\xi,\eta) \tag{2.14}$$

where E is the statistical expectation and the overbar denotes the complex conjugate. In the sequel we will only use processes with mean values equal to zero. Therefore, no distinction is necessary between the autocorrelation r_{ff} and the autocovariance

c_{ff} and we will always use the latter one. The Fourier transform of c_{ff} is given by

$$S_{ff}(u,v) = \iint_{-\infty}^{+\infty} c_{ff}(\xi,\eta)e^{-i(u\xi+v\eta)}\,d\xi\,d\eta \tag{2.15}$$

where S_{ff} is called the spectral density. It can be shown that the spectral density of an arbitrary process is nonnegative

$$S_{ff}(u,v) \geq 0 \ .$$

In general, a stationary process $f(x,y)$ does not have a spectral representation of the form (2.5). Thus, equation (2.15) must be regarded as the basic spectral relation for a stationary process. Given a positive function $S_{ff}(u,v)$ or, equivalently, a positive definite function $c_{ff}(\xi,\eta)$, we can find a stochastic process having $S_{ff}(u,v)$ as spectral density and $c_{ff}(\xi,\eta)$ as covariance function. If the covariance function does not have a Fourier transform it can usually be represented by a Fourier-Stieltjes integral and the inversion of this integral is possible by generalized transform methods. Incidently, this is the method also used for the spectral representation of the process itself.

Let us now consider the convolution (2.3). It would be quite advantageous to have $g(x,y)$ stationary because this would allow us to characterize the simulated field by its first two moments. One condition which secures stationarity of $g(x,y)$ is the following: If the process $f(x,y)$ is wide-sense stationary then the output of the convolution (2.2) will also be stationary. Furthermore, $f(x,y)$ and $g(x,y)$ will be jointly stationary, i.e. the joint statistics of $f(x,y)$ and $g(x,y)$ will be the same as the joint statistics of $f(x+\xi,y+\eta)$ and $g(x+\xi,y+\eta)$. Thus we have

$$c_{fg}(\xi,\eta) = E\{f(x+\xi,y+\eta)\overline{g(x,y)}\}$$

or

$$c_{fg}(\xi,\eta) = c_{ff}(\xi,\eta) ** \overline{h(-\xi,-\eta)} \; . \tag{2.16}$$

Similarly

$$c_{gg}(\xi,\eta) = c_{fg}(\xi,\eta) ** h(\xi,\eta) \tag{2.17}$$

Forming the double transforms $S_{ff}(u,v)$, $S_{fg}(u,v)$, $S_{gg}(u,v)$ of $c_{ff}(\xi,\eta)$, $c_{fg}(\xi,\eta)$, $c_{gg}(\xi,\eta)$, we obtain from (2.6), (2.16) and (2.17)

$$S_{fg}(u,v) = S_{ff}(u,v)\ \overline{H(u,v)}$$

$$S_{gg}(u,v) = S_{fg}(u,v)\ H(u,v)$$

$$S_{gg}(u,v) = S_{ff}(u,v)\,|H(u,v)|^2 \; . \tag{2.18}$$

By use of equation (2.9)

$$S_{gg}(u,v) = 4\pi^2 e^{-2d\sqrt{u^2+v^2}} S_{ff}(u,v) \tag{2.19}$$

Formula (2.19) shows that the spectral density of the simulated process is related in a simple way to the spectral density of the mass density function if $f(x,y)$ is stationary. The importance of this result has already been stressed by Naidu (1968) and Grafarend (1970). It is only valid for the planar approximation while for the spherical case a correction term is necessary. Using the inverse we obtain

$$c_{gg}(\xi,\eta) = \iint_{-\infty}^{+\infty} e^{-2d\sqrt{u^2+v^2}}\ S_{ff}(u,v) e^{i(u\xi+v\eta)} du dv \tag{2.20}$$

and for the variance

$$c_{gg}(0,0) = \iint\limits_{-\infty}^{+\infty} e^{-2d\sqrt{u^2+v^2}} S_{ff}(u,v)\,du\,dv \;. \tag{2.21}$$

There are simple relations between the spectral densities of different first and second order gradients which are e.g. derived in Naidu (1968) and Kubackova (1974).

In order to select a special model we have now to choose a specific spectral density or, equivalently, a specific covariance function. A number of different Gaussian models have been considered by Grafarend (1970). In case of point mass anomalies a model corresponding to white noise seems to be most adequate. In two dimensions it is sometimes called an incoherent process and is defined by the covariance function

$$c_{ff}(x_1,y_1;x_2,y_2) = q(x_1,y_1)\delta(x_2-x_1)\delta(y_2-y_1) \tag{2.22}$$

where δ denotes the Dirac-function.
To get a stationary process we must have

$$q(x,y) > 0 \;. \tag{2.23}$$

Since white noise processes have infinite intensity

$$E\{|f(x,y)|^2\} = \infty$$

it is useful to define the average intensity of the output which in our case is

$$E\{|g(x,y)|^2\} = q(x,y)**|h(x,y)|^2 \tag{2.24}$$

and if $h(x,y)$ takes significant values only in a limited region A near the origin, we get

$$E\{|g(x,y)|^2\} \approx q(x,y)\iint\limits_A |h(x,y)|^2 dx\,dy \tag{2.25}$$

Another model which might be worthwhile considering in our case is a stationary Markov sequence.

So far, we have used only one plane of generating masses. As is apparent from tables 2.1 and 2.2 the combination of different planes would be advantageous for a realistic model of first and second order gradients. Schwahn (1975) has investigated this case and has shown that the autocovariance function of the simulated field can be determined by adding the autocovariance functions of the partial fields if no crosscovariances between the planes are present. This case seems to be applicable in all kinds of model computations. If crosscovariances in z-direction cannot be neglected the full covariance matrix of the partial fields must be used and computations become extremely laborious. The case of a thick sheet of random masses and, as a special case, of a semi-infinite medium has been treated by Naidu (1968).

The practical procedures of generating gravimetric data from a point mass model can only approximate the discussed models. Usually the anomalous masses are allocated to the intersections of a grid in the (x,y)-plane, i.e. the generating field is a two-dimensional array of mass points equidistant along the axes. If the grid covers the infinite plane a good approximation of the above models is always possible. If the grid is only given in a finite region, the situation becomes more difficult because theoretically we will loose stationarity of the simulated function. It can be expected, however, that stationarity will be good enough for all practical purposes if the array of mass points is chosen properly. Obviously, the density of the grid, its extension, and the depth of the generating masses will be important parameters.

3. Error Models.

Two types of statistical error models have been used in the computations: normal models and Markov models. The term normal model will refer to a series of numbers taken from a normal distribution $N\{\mu,\sigma^2\}$ with mean μ and variance σ^2. The term Markov model will be used in connection with Markov sequences of first and second order. The following discussion will be mainly directed towards Markov sequences.

We will assume wide-sense stationarity for these models and thus be able to use some of the results of section 2. Matters are simplified to a certain extent by the fact that we are considering sequences only. They can be viewed as stochastic processes $d(t)$ where t can take integral values only. Furthermore, since we do not assume error correlations between profiles we will only consider the one-dimensional case.

The difference between the normal and the Markov model lies in their correlation characteristics. Normal deviates are uncorrelated, elements of Markov sequences are not. If e_i is an element of the first model, we have

$$E\{e_i\} = \mu_N$$

$$E\{(e_i-\mu_N)(e_i-\mu_N)\} = \sigma_N^2 \tag{3.1}$$

$$E\{(e_i-\mu_N)(e_j-\mu_N)\} = 0 \qquad i \neq j ,$$

where E is again the statistical expectation.

If $d_i = d(t)$ is an element of the second model, we have

$$E\{d_i\} = \mu_M$$

$$E\{(d_i-\mu_M)(d_i-\mu_M)\} = \sigma_M^2 \tag{3.2}$$

$$E\{(d_i-\mu_M)(d_{i+k}-\mu_M)\} = c_k \qquad k = 1, 2, 3, \ldots ,$$

where c_k is the k-th autocovariance with the corresponding correlation

$$r_k = \frac{c_k}{\sigma_M^2} \tag{3.3}$$

Stationarity of the models is apparent from the first two equations of (3.1) and (3.2). Their difference is obvious from the last equation in each group. While any element e_i is independent of any other element e_j, each element d_i of the Markov model does depend on one or more of the previous elements; all d_i have the same univariate distribution but are correlated.

The following presentation of Markov sequences will only cover those characteristics which are interesting for the subsequent computations. Since the presentation will be rather heuristic it may be necessary to consult a more detailed exposition. The introductory texts of Parzen (1962), Yaglom (1973), and Kendall (1976) have been found especially useful.

Markov sequences are distinguished by their order. In an intuitive way the order of the sequence is equivalent to the smallest number of independent values necessary to describe the correlation in the series. Thus, a Markov sequence of first order is of the form

$$d_i = rd_{i-1} + e_i \tag{3.4}$$

where we have only one coefficient r describing the correlation. To obtain the correlation between d_i and d_{i-2} we apply the above formula to d_{i-1} and obtain

$$d_{i-1} = rd_{i-2} + e_{i-1} \; .$$

Inserting this into equation (3.4) we get

$$d_i = r^2 d_{i-2} + re_{i-1} + e_i \; . \tag{3.5}$$

Continuing in this way we see that the correlation between d_i and d_{i+k} is r^k and that we can write

$$d_i = \sum_{k=0}^{\infty} r^k e_{i-k} \; . \tag{3.6}$$

Keeping in mind that r is real $|r| < 1$ and that the correlation function is symmetric we obtain the covariances c_k by

$$c_k = \sigma_N^2 \sum_{\ell=0}^{\infty} r^{\ell} r^{\ell+k}$$

$$c_k = \frac{\sigma_N^2}{1-r^2} r^k . \qquad (3.7)$$

Thus we can express the variance σ_M^2 of the Markov sequence of first order in terms of the variance σ_N^2 by

$$\sigma_M^2 = \frac{\sigma_N^2}{1-r^2} \qquad (3.8)$$

This shows that if $|r|$ is close to one σ_M^2 will be much larger than σ_N^2. Thus a high correlation of sucessive values will produce large amplitudes even if the disturbances are small.

The relation between the spectral density and the covariance function of a stationary Markov sequence can be represented in a form similar to equation (2.15)

$$S_{dd}(w) = \sum_{k=-\infty}^{+\infty} c_{dd}(k) e^{-iwk} \qquad (3.9)$$

where $S_{dd}(w)$ is the spectral density, $c_{dd}(k)$ the covariance function, and where k can take integer values only.

Conversely, we obtain

$$c_{dd}(k) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{dd}(w) e^{iwk} dw . \qquad (3.10)$$

Using equation (3.7) we have for the Markov sequence of first order

$$c_{dd}(\ell) = c_o r^{\ell} \qquad (3.11)$$

where

$$c_o = \frac{\sigma_N^2}{1-r^2} .$$

Since the covariance function is symmetric in k

$$c_{dd}(-k) = c_{dd}(k)$$

we obtain

$$c_{dd}(k) = c_o r^{|k|} \tag{3.12}$$

and using formula (3.9)

$$S_{dd}(w) = c_o \sum_{k=-\infty}^{\infty} r^{|k|} e^{-iwk} . \tag{3.13}$$

After some rearrangement the summation of the series will result in

$$S_{dd}(w) = \sigma_N^2 \frac{1}{|e^{iw}-r|^2} \tag{3.14}$$

or

$$S_{dd}(w) = \sigma_N^2 \frac{1}{(1-2r\cos w+r^2)} . \tag{3.15}$$

Thus, we have determined the spectral density from the covariance function.

A Markov sequence of second order, also called compound Markov sequence or Yule sequence, is characterized by two correlations r_1 and r_2. We can define it by the following equation

$$d_i = -a_1 d_{i-1} - a_2 d_{i-2} + e_i \tag{3.16}$$

where the coefficients a_1 and a_2 are real numbers and satisfy $|a_1| < 1$, $|a_2| < 1$. They are connected to the correlations r_1 and r_2 by

$$\begin{aligned} a_1 &= -\frac{r_1(1-r_2)}{1-r_1^2} \\ a_2 &= -\frac{r_2 - r_1^2}{1-r_1^2} \end{aligned} \tag{3.17}$$

or conversely

$$\begin{aligned} r_1 &= -\frac{a_1}{1+a_2} \\ r_2 &= -a_2 + \frac{a_1^2}{1+a_2} \end{aligned} \tag{3.18}$$

The spectral density of such a sequence is given by

$$S_{dd}(w) = c_2 \frac{1}{|e^{iw}-a_1|^2 |e^{iw}-a_2|^2} . \tag{3.19}$$

Using

$$|e^{iw}-a_1|^2 = (e^{iw}-a_1)(e^{-iw}-a_1)$$

and replacing e^{iw} by z we obtain

$$S_{dd}(w) = c_2 \frac{z^2}{(z-a_1)(1-za_1)(z-a_2)(1-za_2)}$$

and applying partial fractions

$$S_{dd}(w) = \frac{c_2}{(a_1-a_2)(1-a_1a_2)} \left\{ \frac{a_1}{1-a_1^2} \left(\frac{a_1}{z-a_1} + \frac{1}{1-a_1 z}\right) \right.$$

$$\left. - \frac{a_2}{1-a_2^2} \left(\frac{a_2}{z-a_2} + \frac{1}{1-a_2 z}\right) \right\} . \qquad (3.20)$$

The functions $1/(1-a_1z)$ and $1/(1-a_2z)$ are regular in the unit circle. Hence the series expansions contain only nonnegative powers of z. The functions $a_1/(z-a_1)$ and $a_2(z-a_2)$ are regular outside the unit circle and have series expansions

$$\frac{a}{z-a} = \sum_{n=1}^{\infty} a^n z^{-n}$$

for $|z| \geq 1$. Expanding in this way and using equation (3.10) we can determine the covariances

$$c_{dd}(k) = \frac{c_2}{(a_1-a_2)(1-a_1a_2)} \left\{ \frac{a_1}{1-a_1^2} a_1^{|k|} - \frac{a_2}{1-a_2^2} a_2^{|k|} \right\} \qquad (3.21)$$

The variance σ_{MM}^2 of the second-order Markov sequence can be expressed in terms of the variance σ_N^2. We use equation (3.16) in the form

$$e_i = d_i + a_1 d_{i-1} + a_2 d_{i-2}$$

and take statistical expectations on both sides

$$E\{e_i, e_i\} = E\{(d_i+a_1 d_{i-1}+a_2 d_{i-2}),\ (d_1+a_1 d_{i-1}+a_2 d_{i-2})\}$$

$$\sigma_N^2 = \sigma_{MM}^2(1+a_1^2+a_2^2+2a_1 r_1+2a_2 r_2+2a_1 a_2 r_1)\ .$$

Substituting r_1 and r_2 in terms of a_1 and a_2 results in

$$\sigma_{MM}^2 = \sigma_N^2 \frac{1 + a_2}{(1-a_2)\{(1+a_2)^2-a_1^2\}} \tag{3.22}$$

Equation (3.22) can be used to determine the factor c_2 in formula (3.21).

Figures of covariance functions of Markov sequences and of their corresponding spectral densities can e.g. be found in Kendall (1976).

Markov sequences of higher order can be generated in an analogous way. The basic equation

$$d_i = - a_1 d_{i-1} - a_2 d_{i-2} \ \ldots\ldots \ - d_n a_{i-n} + e_i \tag{3.23}$$

has the spectral density

$$S_{dd}(w) = c_2 \frac{1}{|e^{iw}-a_1|^2 |e^{iw}-a_2|^2 \ldots . |e^{iw}-a_n|^2} \tag{3.24}$$

which can again be used to determine the covariance function.

The use of stationary Markov sequences is greatly facilitated by the simple way in which they can be generated. Equations (3.4) and (3.16) can directly be used for this purpose. Subroutines for normal deviates e_i are usually available in program libraries. Otherwise, effective methods to compute normal deviates are e.g. given in Hamming (1962). We then assume d_o

and previous terms to be zero and run the series for a number of terms until the effect of these initial assumptions has become negligible. From this point onwards the series can be regarded as a Markov sequence. To reach this point in a small number of steps the normal deviates e_i should be multiplied by $(\sigma_M^2/\sigma_N^2)^{1/2}$ or $(\sigma_{MM}^2/\sigma_N^2)^{1/2}$ to obtain the right size of the variance for the values d_i. These ratios can be obtained from formulas (3.8) and (3.22).

4. Results

The data used in the following computations have been generated by DMA (Howard, 1976). It had been requested that the variance of the gravity anomalies at ground level should be close to C_o = 1650 mgal^2 and that the variance of the horizontal derivatives of Δg at the same level should be about G_o = 100 E^2. The value of C_o presupposes that a regional part of the gravity field corresponding to an expansion of about degree and order 10 has been subtracted. The field has been generated using a grid of mass points with a spacing of 10' at a depth of 40 km. The total area is $10^\circ \times 15^\circ$, i.e. about 4150 mass points have been used. The mass points have either a positive or a negative mass of constant size or a zero mass. The selection was made according to a normal distribution. More details on the simulation of the data can be found in Howard (1976).

In view of section 2 this model has some restrictions. The depth of the generating masses is about 2.2 times the mass point spacing. Judging from tables 2.1 and 2.2 the high frequency part of the first and second order gradients will be very small and the respective fields will be smooth. Furthermore, the approximation of stationary white noise by a jump process with three possible states (+,-,0) seems somewhat inadequate. Future models should at least have two generating planes, one at a depth of about 40-60 km, the other at about 15-25 km. The dense point spacing is only necessary on the upper plane, the lower one may have a much wider spacing. The point masses should be taken from a normal distribution with the larger variance on the lower plane.

The following results should be seen with these reservations in mind. Thus, statements about accuracy may need some qualification while the conclusions about operational performance should be fairly general. Since the latter was the main objective of this study there was no immediate need for a more elaborate model.

Fig. 4.1 shows the simulated anomalous gravity field at ground level with 20-mgal contour lines. The field shows large variations with the extreme points at about plus and minus 120 mgal and $\overline{C}_o$ = 1741 mgal^2 . The variance $\overline{G}_o$ is at about 96 E^2 .

The formula used to estimate gravity anomalies at ground level from the simulated data at flight level is

$$s = C_{sx} C_{xx}^{-1} x \tag{4.1}$$

where s is the vector of gravity anomalies, called the signal, and x is the vector of simulated data, called observations in the sequel. It should be noted that contrary to section 2 and 3 covariance matrices are denoted by capital letters to simplify comparison with other publications. C_{xx} is the autocovariance matrix of the observations. Since

$$t = x - n \tag{4.2}$$

we have

$$C_{xx} = C_{tt} + C_{nn} \tag{4.3}$$

where t refers to any first or second order gradient used as observation and where n is the error given by one of the error models. C_{sx} contains the crosscovariances between signal and observation, and since s and n are considered to be uncor-

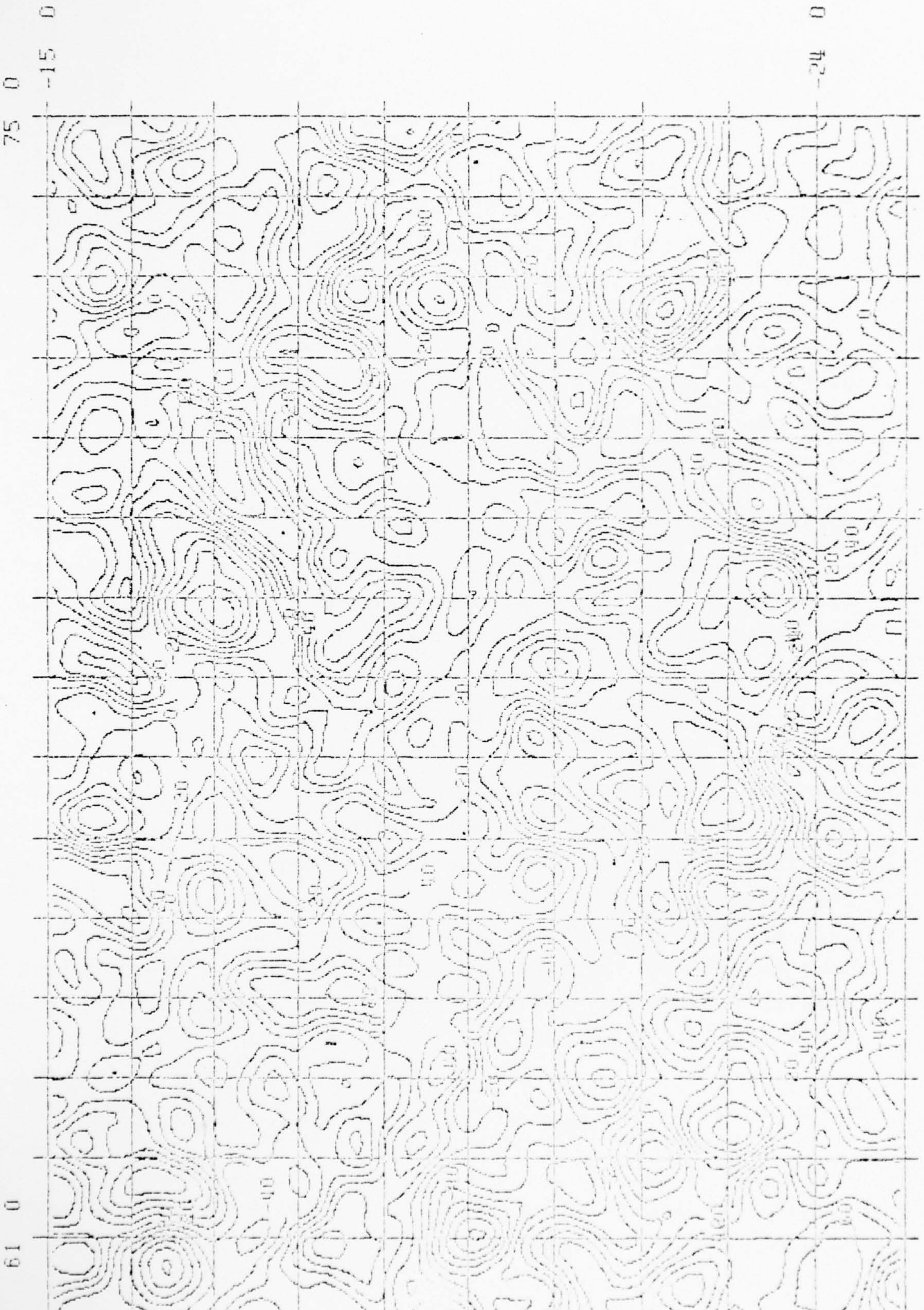

Fig. 4.1 Simulated gravity anomaly field (mgal)

related we have

$$C_{sx} = C_{st} .$$

Both, s and t are quantities of the anomalous field. Their mathematical relation can be expressed by integral equations or infinite series. In formula (4.1) these relations are contained in the covariance matrices, i.e. all infinite operations have been performed on the covariances without approximation. Thus, we have a consistent model for heterogeneous observations and this characteristic property is preserved even if the covariance function is not optimal.

The covariance function used in this report has been described in Schwarz (1976). It has the advantage of numerical simplicity and it agrees well with statistical estimates of the anomalous field. Three of these estimates have been used as essential parameters for the covariance function, namely

$$C_o = 1500 \text{ mgal}^2 \quad \xi = 61 \text{ km} \quad G_o = 111 \text{ E}^2 , \tag{4.4}$$

where C_o is the variance of the gravity anomalies, ξ is the correlation length of the corresponding covariance curve, and G_o is the variance of the horizontal derivatives of the gravity disturbance. All quantities refer to ground level. Fig. 4.2 shows this covariance function as heavy line. The dashed line refers to a covariance function directly derived from the simulated gravity field using a set of 9600 points spaced at 7!5 intervals. Its essential parameters are

$$\overline{C}_o = 1741 \text{ mgal}^2 \quad \overline{\xi} = 52 \text{ km} \quad \overline{G}_o = 96 \text{ E}^2 . \tag{4.5}$$

It will be used later on to determine the influence of wrong assumptions in the covariance function on the estimation.

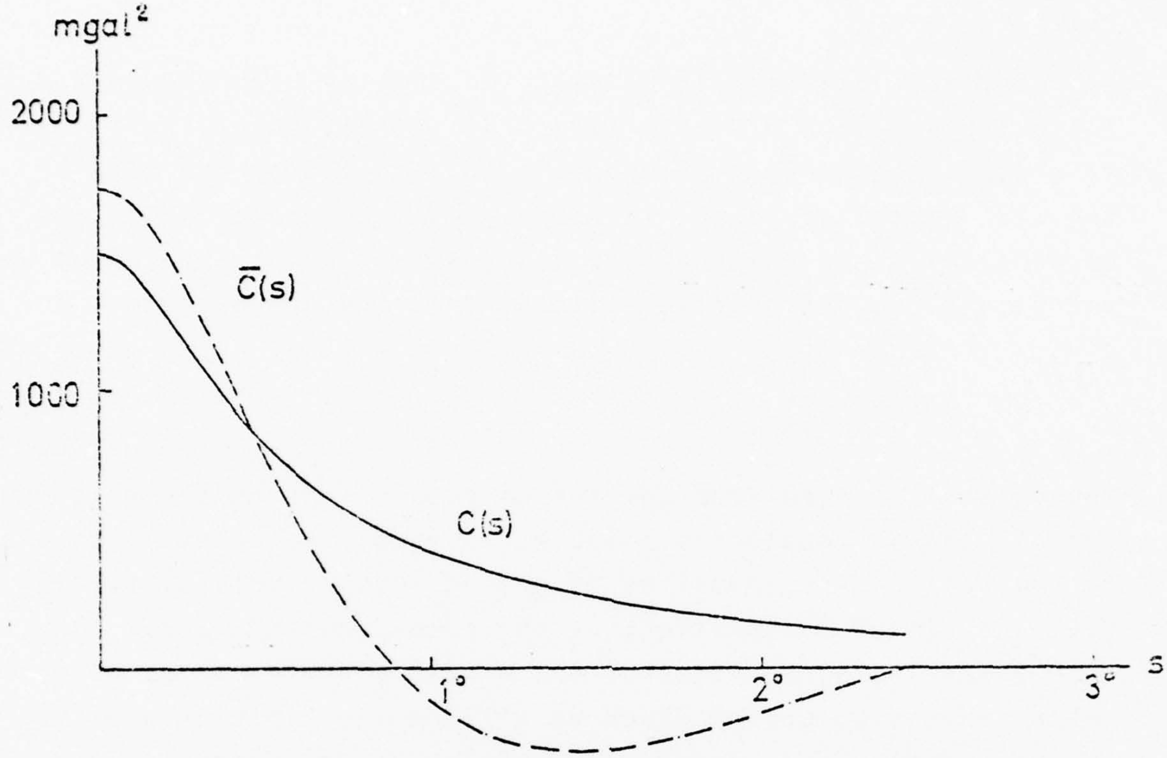

Fig. 4.2 Gravity anomaly covariance functions used in the computations.

The observations at flight level (10 km) have been generated for a flight speed of 500 knots and an integration interval of 10 sec. Thus, the separation of data points is a constant 2.6 km. The profiles are parallel and run in east-west direction. They cover an area of $10^\circ \times 15^\circ$. Generally, the profile spacing is 1°, except in a strip of $2^\circ \times 15^\circ$ where it is 20'. The choice of the east-west profiles was suggested by the favourable error behaviour in this direction (Meissl, 1970) and because simulations are especially simple with such an arrangement. There are no basic changes, however, if an arbitrary direction is used as long as the profiles remain parallel and the data points have a constant separation.

It has been shown in Schwarz (1976) that the accuracy of the estimation is mainly dependent on the point configuration and not so much on the number of observations used. On the other hand, it has been pointed out that for reasons of reliability all available data should be used. To combine both viewpoints the following approach has been taken. An optimal point configuration is chosen depending on the profile spacing and on the size of the mean anomalies to be estimated. With the above conditions on parallelism of the profiles and constant point separation we can move along the profiles without changing the operator R

$$R = C_{sx} C_{xx}^{-1} . \tag{4.6}$$

Moving step by step from one set of data points to the next we use all the infromation available. In each step we estimate one or several gravity anomalies at ground level simply by multiplying a set of observations by the predetermined matrix R . When advancing along the profiles we get a whole series of estimation points on ground which we will call estimation profiles in the sequel. Thus, even long profiles can be processed very fast.

Fig. 4.3 shows the principle of the moving operator R for the estimation of one gravity anomaly in each step using two

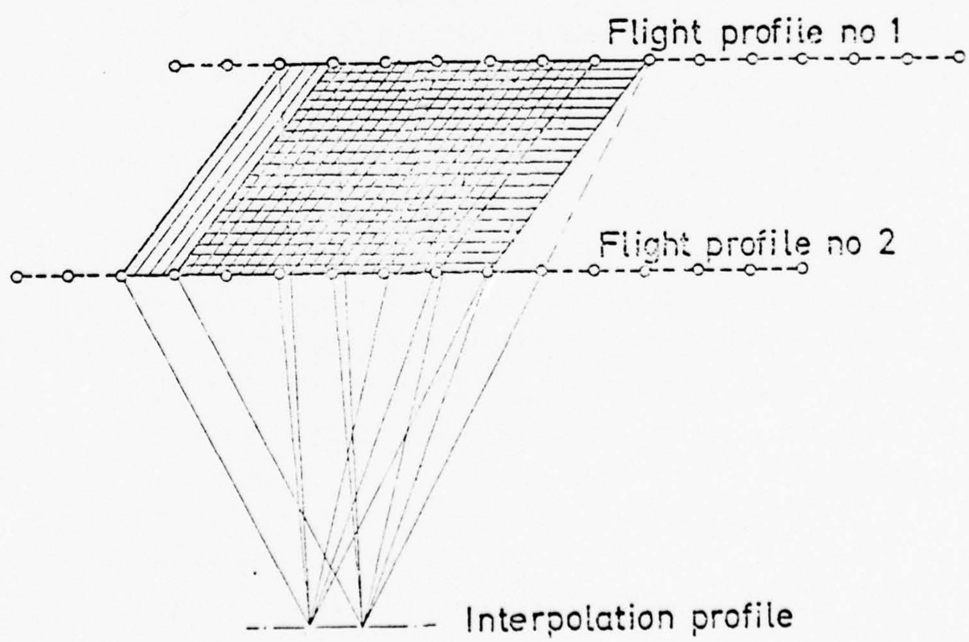

Fig. 4.3 Estimation by a moving operator

profiles of observations. Obviously, the addition of more estimation or more observation profiles does not change the basic procedure. We can estimate point as well as mean gravity anomalies by this method. It was found, however, that mean anomalies should not be determined by a mean anomaly covariance function. The smoothing properties of this function will cause a considerable loss of information. Sünkel (1977) has derived detailed formulas to estimate the loss of accuracy for a given block size and a given covariance function. To avoid such a loss the following method has been adopted to determine mean values. Depending on the size of the block 3 to 5 estimation profiles are used at ground level and gravity anomalies are estimated along these profiles at the same rate as taken at flight level. Thus, in our case the separation of the point gravity anomalies along the estimation profiles is 2.6 km. All values inside a block are averaged to obtain the mean value. In this way the accuracy of the point estimation is maintained and we obtain an averaging procedure which is linked in a simple way to the estimation method.

It has been shown in Schwarz (1976) that Δg, T_{rr}, and $T_{r\phi}$ are the most important observations when estimating gravity anomalies from east-west profiles. These three measurements have therefore been simulated in each data point at flight level. To give an idea of the accuracy of point estimation, fig. 4.4 shows about 3° of an estimation profile obtained from flight profiles spaced at 20'. The standard error for the Δg-observations is $\pm$ 1 mgal, that of the gradiometer observations $\pm$ 1 E . The normal error model has been used. The heavy line shows the exact profile of the simulated field at ground level while the gravity anomalies estimated from the corrupted data are represented by dots. The agreement is very good with a standard error of about $\pm$ 3.3 mgal for the point estimation. To determine how much of this error is due to interpolation and how much to downward continuation, we have determined an interpolation profile from the same data but this time located directly below one of the flight profiles. The standard error reduces to $\pm$ 1.96 mgal. Thus

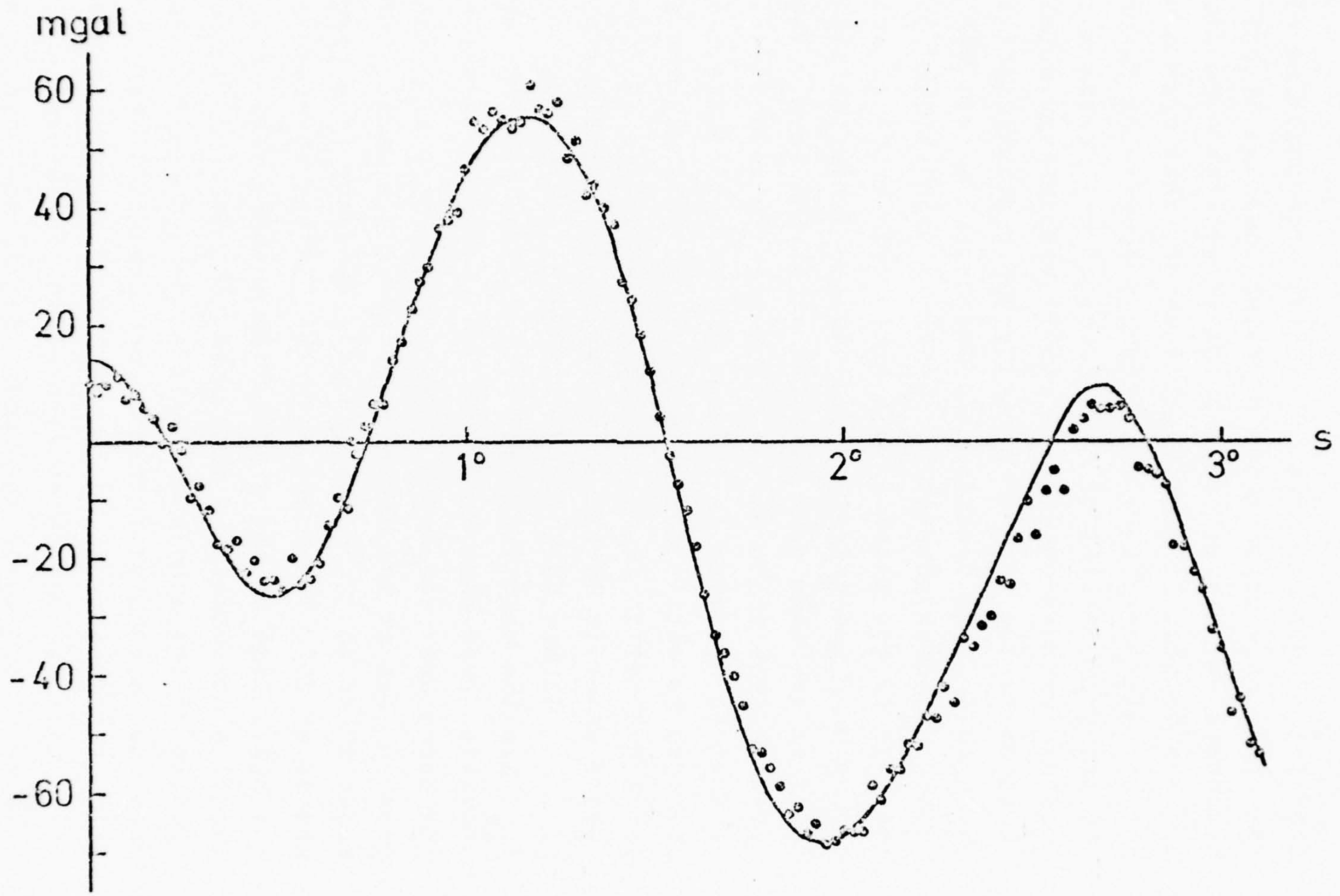

Fig. 4.4 Comparison of simulated and estimated gravity profile.

we can say that with the above configuration a measuring error of $\pm$ 1 mgal at flight level will be amplified by downward continuation to about $\pm$ 2 mgal and when interpolated between two profiles 20' apart to about $\pm$ 3 mgal . A dense profile spacing will therefore improve the accuracy considerably.

Results for mean anomalies are given in table 4.1. The same error model as above has been used. The number of mean values estimated in each case is given and although the sample size is small in one case, the internal consistency of the different values is good. For comparison results obtained in the corresponding accuracy analysis (Schwarz, 1976) are also shown. In general, the figures agree well. But the accuracy estimates seem to be somewhat smaller for block sizes below 30'x30' and larger for block sizes above 30'x30'.

Block size	Profile spacing	Number of values	Simulation study (mgal)	Accuracy study (mgal)
15'x15'	20'	232	$\pm$ 2.2	$\pm$ 1.7
30'x30'	20'	112	$\pm$ 1.8	$\pm$ 1.7
1°x1°	20'	26	$\pm$ 1.3	$\pm$ 1.7
1°x1°	1°	78	$\pm$ 4.6	$\pm$ 5.6

Table 4.1 Accuracy for different block sizes as obtained from simulation and from accuracy studies.

The influence of different error models has been studied for a number of cases. Table 4.2 gives results for mean values of 15'x15' , a profile spacing of 20', and a standard error of the gradiometer observations of $m_{gg} = \pm 1$ E . Normal and Markov models have been computed for different variances σ_N^2 . The Markov model is of second-order and has the correlations $r_1 = 0.73$ and $r_2 = -0.33$.

Block size	Profile spacing	Number of values	σ_N (mgal)	Estimation error m_s in mgal	
				Normal	Markov
15'x15'	20'	232	$\pm$ 1.0	$\pm$ 2.2	$\pm$ 2.3
			$\pm$ 2.2	$\pm$ 2.8	$\pm$ 3.7
			$\pm$ 5.0	$\pm$ 4.5	$\pm$ 10.7

Table 4.2 Comparison of error models

The results are remarkable because they show a significant difference in the general behaviour of the two error models once the variance has reached a certain size. While there is not much difference on the $\pm$ 1-mgal level because interpolation and downward continuation contribute the largest part to the error budget, the difference is quite obvious on the $\pm$ 5-mgal level. For the normal model the estimation error at ground is smaller than at flight level while for the Markov model it is more than twice the size. In the first case the error in Δg is controlled by the accurate gradiometer data. In the second case the variance σ_N^2 of the generating process is enlarged by a factor of almost 3 due to the correlations in the Markov sequence, see formula (3.22). This raises the standard error of $\sigma_N = \pm 5$ mgal to about $\sigma_{MM} = \pm 8.5$ mgal . The actual estimation error $m_s = \pm 10.7$ mgal is even larger because signal and noise cannot be separated as well as in case of the normal model. Thus, results will deteriorate considerably if the errors in the Δg-values at flight level are correlated and if their variance is not extremely small.

The situation is even worse if the data are corrupted by systematic errors. Only a few examples have been computed and more representative studies are necessary. But a systematic influence at the 5-mgal level may already turn the estimation results useless for geodetic purposes.

Finally, some investigations have been made on the influence of wrong assumptions in the covariance function. So far, only the covariance function $C(s)$ with the essential parameters (4.4) has been used. It will now be compared to the covariance function $\overline{C}(s)$ determined from the simulated data with the parameter set (4.5). To have an easy distinction the latter will be labeled 'correct covariance function'. Results are given in table 4.3. As could be expected the correct covariance func-

Block size	Profile spacing	Estimation error m_s in mgal	
		$C(s)$	$\overline{C}(s)$
15'x15'	20'	± 2.2	± 2.1
30'x30'		± 1.8	± 1.7
1°x 1°		± 1.3	± 1.0
1°x 1°	1°	± 4.6	± 6.2

Table 4.3 Comparison of different covariance functions.

tion $\overline{C}(s)$ gives slightly better results for all block sizes when a profile spacing of 20' is used. The differences are not large, however, and we can conclude that the choice of the covariance function does not affect the estimation very much as long as the essential parameters are reasonably close to the empirical values. This confirms results published in Moritz (1976). It should be kept in mind, however, that these results are valid for isotropic covariance functions only and may not hold in more difficult cases.

The last line of table 4.3 which shows the 1°x1° mean for a profile spacing of 1° apparently contradicts the above conclusion. In this case the C(s)-function gives much better results. The most probable explanation is that the data are not dense enough to get a useful estimate from the $\overline{C}(s)$-function. The correlation length $\overline{\xi}$ is smaller than one half of the profile spacing and it has been found that results may become very poor in such a case. Thus, estimation with a non optimal covariance function may in some cases give better results.

5. Brief Description of Programs.

The programs and a sample computation have been added to the report. Since detailed comments are given in the subroutines we will present only a brief survey of the programs and point out a few restrictions in the present version.

Basically, the program computes point or mean anomalies at ground level from measurements at flight level. Terrestrial and satellite observations may also be added. It has been assumed that some preprocessing of the airborne data has taken place, and that besides 5 second-order gradients the value of Δg is available in each observation point. In the present form a total of 7 observations per point can be read but fewer are possible. All data should be available on a direct access file. The subroutine ZINF selects the observation profiles needed for the estimation of a specific point. The subroutine DATS reads these profiles and deletes those measurements which are not needed. In this way the core storage requirements can be kept relatively small. Furthermore, the simulated data from the file are corrupted by one of the error models in DATS. This part of the program must be deleted when handling real data.

The number of estimation profiles for a specific mean value is fixed in VMEAN. We have used 5 estimation profiles for blocks larger than 40'x40', 3 profiles for blocks larger than 5'x5' and smaller than 40'x40', and 1 profile for blocks of 5'x5' and point values. If more than 5 estimation profiles are used the corresponding DIMENSION-statements have to be changed. The covariance matrices are set up in the subroutine PLAY which uses the subroutine COVAX to determine the individual covariances. The estimation according to formula (4.1) is done by the subroutines FIX and SCAN. The subroutine COMPA compares these estimated values to the true values of the simulation which are again stored on a direct access file. The error computations are also performed by this subroutine. Again, this part of the program must be deleted when handling real data.

Three of the subroutines used are not listed. The subroutine COVAX has been published in (Tscherning, 1976). Three corrections which have been communicated by the author are listed. The subroutines DSINV and GAUSS are available in the Fortran Scientific Subroutine Package which can be found in IBM publications. The first one inverts a double precision matrix stored in a one-dimensional array. The second one generates normal deviates.

In its present form the program accepts flight profiles in east-west direction with 4200 observations each. This corresponds to a profile length of 16° if the flight speed is 500 knots and 6 measurements are taken at each point. From a total of 11 profiles a maximum of 5 profiles is selected for each estimation. Since the storage requirements are strongly dependent on the length of the profiles, a subdivision of longer profiles might be considered. Because of the use of the direct access file an increase in the number of profiles is not critical as long as the maximum number of profiles in the core storage is kept to 5.

The speed of the data processing depends mainly on the number of flight profiles used in each estimation and only to a smaller extent on the block size of the mean value. As an example mean values covering an area of $6^\circ x 13^\circ$ have been computed using the data configuration and the covariance function given in section 4. With an average of 3 flight profiles for each estimation about 30 seconds of CPU time are needed to determine 75 blocks of $1^\circ x 1^\circ$; for 5 profiles the time required increases to about 50 seconds. If we estimate 300 blocks of 15'x15' instead, another 10 % must be added to the time estimates. Thus, even large amounts of data can be processed in a relatively short time.

6. Conclusions

The simulation studies presented in this report show that least-squares collocation offers an adequate model to estimate gravity anomalies from airborne gradiometer measurements. The procedure is simple numerically and allows to handle large amounts of data with regular requirements on core storage and small demands on computer time.

The deviations of the estimated gravity anomalies from their true values agree well with the estimates obtained from corresponding accuracy studies (Schwarz, 1976). It should be noted, however, that correlated errors in the measurements will strongly influence the accuracy of the results. A second-order Markov sequence has been used to model the error process along the profiles. Depending on the size of the correlations and the variance of the process, the mean-square errors will more than double as compared to the uncorrelated case. Similarly, a bias in the data will impair the accuracy of the results considerably.

The simulation of gravimetric quantities from a point mass model is considered in the spectral domain and conclusions are drawn with respect to the resulting fields. In order to represent adequately regional variations of the gravity field as well as the local behaviour of the second-order gradients the medium and the high frequency part of the spectrum must be modeled equally well. This can only be achieved by using several planes of generating masses at different depths. In many cases a model with two planes may already be sufficient.

Acknowledgements.

The simulated data used in the test computations and the computer plot shown in fig.4.1 have been provided by DMA, St. Louis. The cooperative support of several persons in this agency is gratefully acknowledged. L. Krieg of the Department of Geodetic Science at the Ohio State University gave valuable advice and assistance in setting up the direct access files on the IBM 370/168 system. Computer time has been made available by the Instruction and Research Computer Center of the Ohio State University.

REFERENCES

E. Grafarend (1970): The Gaussian Structure of the Gravity Field. Studia geophysica et geodaetica, vol 14, pp 159-167.

E. Grafarend (1976): Geodetic Applications of Stochastic Processes. Physics of the Earth and Planetary Interiors, vol 12, pp 151-179.

R.W. Hamming (1962): Numerical Methods for Scientists and Engineers. New York.

H. Howard (1976): Simulated Gravity Field Data for Computer Program Testing (Models 306 and 307). RDGG unpublished manuscript, DMAAC, St. Louis.

M. Kendall (1976): Time-Series. Second edition, London and High Wycombe.

L. Kubackova (1974): Autocovariance Function and Spectral Density of the Anomalous Gravitational Field, Transformed by an Integral Operator of the Convolution Type. Studia geophysica et geodaetica, vol 18, pp 329-338.

P. Meissl (1970): Probabilistic Error Analysis of Airborne Gravimetry. OSU Report no 138, Columbus, Ohio.

H. Moritz(1975): Combination of Aerial Gravimetry and Gradiometry. OSU Report no 223, Columbus, Ohio.

H. Moritz (1976): Covariance Functions in Least-Squares Collocation. OSU Report no 240, Columbus, Ohio.

P. Naidu (1968): Spectrum of the Potential Field Due to Randomly Distributed Sources. Geophysics, vol 33, pp 337-345.

E. Parzen (1962): Stochastic Processes. San Francisco.

A. Papoulis (1968): Systems and Transforms with Applications in Optics. New York.

W. Schwahn (1975): Eine allgemeine Formulierung der Auto- und Kreuzkorrelationsfunktionen eines beliebigen statistischen Potentialfelds in einem kartesischen Koordinatensystem. Gerlands Beiträge zur Geophysik, vol 84, pp 143-154.

K.P. Schwarz (1976): Geodetic Accuracies Obtainable from Measurements of First and Second Order Gravitational Gradients. OSU Report no 242, Columbus, Ohio.

K.P. Schwarz (1977): Capabilities of Airborne Gradiometry for Gravity Estimation. Bolletino di Geodesia e Scienze Affini (in press).

H. Sünkel (1977): Die Darstellung geodätischer Integralformeln durch bikubische Spline-Funktionen. Mitteilungen der geodätischen Institute der TU Graz (in press).

C.C. Tscherning (1976): Covariance Expressions for Second and Lower Order Derivatives of the Anomalous Potential. OSU Report no 225, Columbus, Ohio.

A.M. Yaglom (1973): An Introduction to the Theory of Random Functions. Dover, New York.

OSU Report ... Reports of the Department of Geodetic Science, The Ohio State University.

Appendix A: Computer Programs

```
C
C     SIMULATION STUDY AIRBORNE GRADIOMETRY
C
C     THE PROGRAM COMPUTES POINT AND MEAN GRAVITY ANOMALIES ON GROUND FROM
C     AERIAL GRAVITY AND GRADIOMETRY MEASUREMENTS AND FROM GEOID INFORMATION
C     PROVIDED BY SATELLITE ALTIMETRY. THE COMPUTED VALUES ARE COMPARED TO
C     'TRUE' VALUES GENERATED BY A MASS ANOMALY MODEL. DEVIATIONS ARE GIVEN
C     FOR ALL COMPUTED VALUES AND THE EFFECT OF DATA PERTURBATIONS OF VARIOUS
C     KINDS CAN BE DETERMINED. AT THE PRESENT STAGE THE PROGRAM IS RESTRICTED
C     TO EAST-WEST PROFILES
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(15),FB(15),FC(16)
     1,TRANS(12),SR(5),CO,COO,VVM,SHI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
      COMMON /CMCOV/CI(12),CR(51),SIGMA0(300),SIGMA(300),KI(25),N1,LOCAL
      COMMON /SHT/ ZZ(3,6)
      DIMENSION AA(46,46),BB(23,46),BBB(23,23),GD(10),GDD(10),GN(10)
      DIMENSION AAA(1100),RR(612,6)
      EQUIVALENCE (AA(1,1),AAA(1))
      DATA GM,RE/3.98D14,6371.0D3/
     *,D0,D1,D2/0.0D0,1.0D0,2.0D0/,PI/3.14159265350D0/
C
C     RE     MEAN RADIUS OF THE EARTH
C     GM     PRODUCT OF THE GRAVITATIONAL CONSTANT AND THE MASS OF THE EARTH
C     THE COMMON AREA /TICK/ IS USED TO TRANSFER DATA FROM THE MAIN
C     PROGRAM TO ALL OTHER SUBROUTINES. THE COMMON AREA /CMCOV/ IS USED TO
C     TRANSFER DATA FROM THE MAIN PROGRAM VIA THE SUBROUTINE PLAY TO THE
C     SUBROUTINE COVAX.
C
      CO = 1.0D0/57.2957795100
      COO = CO/60.
      MA = 46
      MB = MA/2
      DO 110 I=1,MA
      DO 110 J=1,MA
      DO 110 K=1,MB
      AA(I,J) = 0.
  110 BB(K,I) = 0.
      DO 111 I=1,612
      RR(I,3) = 100.
      DO 111 J=1,2
      RR(I,J) = 0.
      DO 111 K=4,6
  111 RR(I,K) = 0.
C
C     D A T A   I N P U T
C
C     INPUT COVARIANCE FUNCTION
C
  100 READ (5,9) S,A,KI(5),K2,K3,N,LOCAL
    9 FORMAT (2D14.7,4I5,L2)
      IF (N.NE.0) GO TO 101
      LOCAL = .TRUE.
      N = 2
  101 N1 = N+1
      IF(.NOT.LOCAL)READ(5,13)(SIGMA0(I), I = 1, N1)
```

```
   13 FORMAT(12F6.2)
      IF (.NOT.LOCAL)WRITE(6,7)(SIGMAO(I), I = 1, N1)
    7 FORMAT(' EMPIRICAL ANOMALY DEGREE-VARIANCES IN UNITS OF MGAL**2:',
     */,25(12F6.2/))
C
C        S      SQUARE OF THE RATIO BETWEEN RADIUS OF THE BJERHAMMER SPHERE AND
C               MEAN RADIUS OF THE EARTH
C        A      CONSTANT FACTOR OF GRAVITY ANOMALY DEGREE-VARIANCE MODEL
C        KI(5)  NUMBER OF DEGREE-VARIANCE MODEL AS SPECIFIED IN:
C               TSCHERNING(1975):COVARIANCE EXPRESSIONS FOR SECOND AND
C               LOWER ORDER DERIVATIVES OF THE ANOMALOUS POTENTIAL
C               OSU REPORT NO 225
C               POSSIBLE NUMBERS: 1,2,3
C        K2,K3  INTEGER K2 AND K3 IN FORMULA (17) IN TSCHERNING(1975).
C        N      DEGREE VARIANCES UP TO AND INCLUDING DEGREE N ARE EITHER SET
C        LOCAL  EQUAL TO ZERO OR REPLACED BY EMPIRICAL DEGREE VARIANCES. IN THE
C               FIRST CASE THE LOGICAL VARIABLE LOCAL MUST BE .TRUE. IN THE
C               SECOND CASE .FALSE. THE EMPIRICAL GRAVITY ANOMALY DEGREE
C               -VARIANCES MUST BE READ IN UNITS OF MGAL SQUARED.
C
C     INPUT FLIGHT PROFILES AND MEASUREMENTS
C
      READ (5,1) NN,VMM,VI,KKL
    1 FORMAT (I5,2D12.2,I5)
      DO 2 I=1,NN
      READ (5,3) B(I),C(I),D(I),GN(I),GD(I),GDD(I),KK(I),KKK(I),
     1(ID(I,J),J=1,7)
    3 FORMAT (6D7.2,I5,8I3)
    2 D(I) = D(I)*1000.
C
C        NN     NUMBER OF FLIGHT PROFILES OBSERVED
C        KK     NUMBER OF POINTS PER PROFILE
C        KKK    NUMBER OF MEASUREMENTS PER POINT
C        KKL    NUMBER OF RECORDS TO BE READ FROM EACH FLIGHT FILE
C        VMM    MEAN VELOCITY OF AIRCRAFT IN KNOTS PER HOUR
C        VI     MEAN INTERVAL BETWEEN OBSERVATIONS IN SECONDS OF TIME
C        B      LATITUDE OF INITIAL POINT
C        C      LONGITUDE OF INITIAL POINT
C        D      ALTITUDE OF INITIAL POINT
C        ID     NUMBERS SPECIFYING MEASUREMENTS AS IN TSCHERNING (1975)
C               POSSIBLE NUMBERS: 1,3,5,10 TO 14
C               SEQUENCE OF NUMBERS ASCENDING.IF LESS THAN 7 MEASUREMENTS,
C               THE REMAINING POSITIONS MUST BE FILLED BY ZEROS.
C        Z      MATRIX STORING THE OBSERVATIONS.EACH ROW CONTAINS ONE
C               PROFILE.MEASUREMENTS ARE STORED POINT BY POINT IN THE
C               SEQUENCE DESCRIBED BY ID.POINTS ARE ORDERED ACCORDING
C               TO ASCENDING LONGITUDE,PROFILES ACCORDING TO DESCENDING
C               LATITUDE.
C        GN     VARIANCE OF GEOIDAL UNDULATIONS IN METER SQUARED
C        GD     VARIANCE OF DELTA G IN MGAL SQUARED
C        GDD    VARIANCE OF SECOND DERIVATIVES IN EU SQUARED
C
C     INPUT INTERPOLATION PROFILES AND MEAN ANOMALY BLOCKS
C
      READ (5,5) IDI,MM,IFI,IT2,VM,SMI
    5 FORMAT (4I5,2F10.0)
      NNN = NN+1
      NM = NN+MM
      KE = 0
      DO 27 I=NNN,NM
      READ (5,11) B(I),C(I),D(I),KK(I)
   11 FORMAT (3D7.2,I3)
```

```
   27 KE = KE+KK(I)
      ITZ = ITZ*4
      READ (IFI,32) (Z(1,J),J=1,ITZ)
   32 FORMAT (4A4)
      K = 0
      DO 31 I=1,ITZ,4
      K = K+1
      RR(K,4) = Z(1,I)
      RR(K,5) = Z(1,I+1)
   31 RR(K,6) = Z(1,I+2)
      IFI = ITZ/4
C
C        IDI    -1  POINT VALUE
C                1  MEAN VALUES
C        MM     NUMBER OF INTERPOLATION PROFILES - POINT OR MEAN VALUES
C        KK     REQUIRED NUMBER OF INTERPOLATION POINTS PER PROFILE - POINT
C               OR MEAN VALUES
C        IFI       FILE NUMBER WHERE MEAN VALUES OF TRUE MODEL ARE STORED
C        ITZ    NUMBER OF MEAN VALUES STORED IN FILE 'IFI'
C        VM     SIZE OF MEAN ANOMALY BLOCK IN MINUTES OF ARC. MUST BE ZERO
C               FOR POINT VALUES.
C        SMI    MEAN PROFILE SPACING IN MINUTES OF ARC
C        ALL OTHER QUANTITIES ARE AS DEFINED FOR THE FLIGHT PROFILES
C
      IF (IDI) 24,22,23
   22 WRITE (6,25) IDI
   25 FORMAT (///2X,'PARAMETER IDI CANNOT BE ZERO.EXECUTION INTERRUPTED'
     1,///)
      GO TO 301
   24 VM = 0.
C
C     D A T A   C O N T R O L
C
   23 WRITE (6,14)
   14 FORMAT (/50X,'D A T A   C O N T R O L',//50X)
      WRITE (6,12) S,A,KI(5),K2,K3,N,LOCAL
   12 FORMAT (' PARAMETERS SPECIFYING THE DEGREE-VARIANCE MODEL',//,10X,
     1'S =',F14.6,/10X,'A =',F13.4,/10X,'KI(5) =',I4,/10X,'K2 =',I7 ,/10
     2X,'K3 =',I7 ,/10X,'N =',I3 ,/10X,'LOCAL =',L4/)
      WRITE (6,20) VMM,VI,VM,VM,SMI
   20 FORMAT (//2X,'MEAN VELOCITY OF AIRCRAFT IN KNOTS PER HOUR:',F9.2,/
     12X,'MEAN INTERVAL BETWEEN MEASURING POINTS IN SECONDS OF TIME:',F7
     2.2,/2X,'SIZE OF MEAN ANOMALY BLOCK:',F5.0,'*',F5.0,' MINUTES',/2X,
     3'MEAN PROFILE SPACING:',F7.2,' MINUTES',///)
      DO 19 I=1,NN
      WRITE (6,18) I
   18 FORMAT (2X,'FLIGHT PROFILE NO',I3)
      WRITE (6,15) B(I),C(I),D(I),KK(I)
   15 FORMAT (2X,'INITIAL POINT OF PROFILE   LATITUDE:',F7.2,'   LONGITU
     1DE:',F7.2,'  ALTITUDE:',F7.0,/2X,'NUMBER OF POINTS IN PROFILE:',I4
     2)
   19 WRITE (6,16) GN(I),GD(I),GDD(I),(ID(I,J),J=1,7)
   16 FORMAT (2X,'VARIANCE  OF GEOID UNDULATIONS:',F7.2,'   OF DELTA G:'
     1,F7.2,'   OF SECOND ORDER DERIVATIVES:',F7.2,/2X,'SPECIFICATION NU
     2MBERS:',7I3/)
      WRITE (6,951)
      WRITE (6,951)
  951 FORMAT (2X/)
      DO 4 I=NNN,NM
      IR = I-NNN+1
      WRITE (6,17) IR
   17 FORMAT (2X,'INTERPOLATION PROFILE NO',I3)
```

```
      D(I) = D(I)*1000.
    4 WRITE (6,15) B(I),C(I),D(I),KK(I)
C
C     C O M P U T A T I O N S
C
      WRITE (6,951)
      WRITE (6,951)
      WRITE (6,952)
  952 FORMAT (/50X,'C O M P U T A T I O N S',//)
C
C     COVARIANCE MODEL
C
      RB2 = RE*RE*S
      KI(3) = K2
      KI(4) = K3
      LMODEL = N.EQ.0
      CI(8) = A*RB2*1.0D-10
      CI(10) = S
      CALL COVAX
C
C     DETERMINATION OF FLIGHT PROFILES NECESSARY FOR INTERPOLATION
C     NUMBERED ICKE
C
      DO 30 I=1,NM
      B(I) = B(I)*CO
   30 C(I) = C(I)*CO
      VVM = VMM*VI/3600.
      CALL VMEAN (VM)
      IF (VM) 301,28,28
   28 ICKE = NNN
      VM = VM*COO
      VVM = VVM*COO
      IMM = 1
  300 CALL ZINF (ICKE,VM)
      IF (ITZ) 302,29,29
   29 IC = ICKE-NN
      WRITE (6,953) IC,(TRANS(J),J=1,NJ)
  953 FORMAT (//2X,'INTERPOLATION PROFILE NO',I3,' USES FLIGHT PROFILES
     1NOS',5F5.0,//)
C
C     COMPUTATION OF AUTOCOVARIANCE MATRIX OF OBSERVATIONS CXX
C
  303 CALL PLAY (AA,BB,-1,MA,MB)
      MU = 0
      DO 230 I=1,NJ
  230 MU = MU+KKP(I)*KKKP(I)
      JLL = NJ+MV
      JJ = 0
      DO 232 I=1,NJ
      KKKK = KKP(I)
      DO 232 II=1,KKKK
      DO 232 J=1,7
      IF (IDP(I,J)-1) 233,234,235
  233 CONTINUE
      GO TO 232
  234 JJ = JJ+1
      IN = TRANS(I)
      AA(JJ,JJ) = AA(JJ,JJ)+GN(IN)
      GO TO 232
  235 IF (IDP(I,J)-3) 233,323,324
  323 JJ = JJ+1
      IN = TRANS(I)
```

```
      AA(JJ,JJ) = AA(JJ,JJ)+GD(IN)
      GO TO 232
  324 JJ = JJ+1
      IN = TRANS(I)
      AA(JJ,JJ) = AA(JJ,JJ)+GDD(IN)
  232 CONTINUE
C
C     INVERSION OF CXX
C
      CALL ORDER (AA,MA,MU,-1)
      SDF = 1.E-50
      CALL DSINV (AAA,MU,SDF,IER)
      IF (IER) 955,956,955
  956 WRITE (6,973)
  973 FORMAT (//2X,'INVERSION OF COVARIANCE MATRIX OKAY',//)
      GO TO 248
  955 WRITE (6,974)
  974 FORMAT (//2X,'INSTABILITIES OCURRED DURING THE EXECUTION OF THE IN
     1VERSION PROGRAM. EXECUTION INTERRUPTED.',//)
      GO TO 302
  248 CALL ORDER (AA,MA,MU,1)
      IF (JJ-MU) 236,237,236
  236 WRITE (6,249) JJ,MU
  249 FORMAT (/20X,'NUMBER OF ROWS IN CXX-MATRIX ERRONEOUS JJ=',I3,'MU='
     1,I3/)
  237 DO 231 I=1,MU
      DO 231 J=1,MU
  231 AA(J,I) = AA(I,J)
      MUM = MV
C
C     COMPUTATION OF PREDICTION MATRIX
C
      CALL PLAY (AA,BB,0,MA,MB)
      DO 239 J=1,MU
      DO 240 I=1,MUM
      SUM = 0.
      DO 240 JJ = 1,MU
      SUM = SUM+BB(I,JJ)*AA(JJ,J)
  240 BBB(I,1) = SUM
      DO 241 II=1,MUM
  241 AA(II,J) = BBB(II,1)
  239 CONTINUE
C
C     COMPUTATION OF SIGNAL (POINT OR MEAN VALUES)
C
      IZI = -1
      CALL DATS (KKL,GN,GD,GDD,IZI)
      CALL SCAN (AA,MA,IMM,VM,RR,ICKE)
      DO 242 I=1,MUM
      DO 242 J=1,MUM
      SUM = 0.
      DO 250 JJ=1,MU
  250 SUM = SUM+AA(I,JJ)*BB(J,JJ)
  242 BBB(I,J) = SUM
C
C     COMPUTATION OF COVARIANCE MATRIX OF SIGNAL CSS
C
      CALL PLAY (AA,BB,1,MA,MB)
      DO 307 I=1,MUM
      DO 307 J=1,MUM
  307 AA(J,I) = AA(I,J)
C
```

```
C     COMPUTATION OF ERROR COVARIANCE MATRIX OF SIGNAL ESS
C
      DO 243 I=1,MUM
      DO 243 J=1,MUM
  243 BBB(I,J) = AA(I,J)-BBB(I,J)
      ICKE = ICKE+1
      IF (ICKE-NM) 300,300,302
  302 ICKE = NNN
C
C     P R I N T   R E S U L T S
C
      DO 306 I=1,KE
  306 RR(I,1) = RR(I,1)/CO
      DO 308 I=NNN,NM
  308 B(I) = B(I)/CO
      SUM = 0.
      DO 310 I=1,MUM
  310 SUM = SUM+BBB(I,I)
      IF (MUM-1) 311,311,312
  311 SUMS = 1
      GO TO 313
  312 SUMS = MUM*(MUM-1)
  313 SMI = DSQRT (SUM/SUMS)
      CALL COMPA (RR,KE,SUM,SUMS)
      VM = VM/COO
      IF (IDI) 968,969,969
  969 WRITE (6,8) VM,VM
    8 FORMAT (//40X,'MEAN VALUES FOR BLOCKS OF',F5.0,' *',F5.0,' MINUTES
     1',//)
      GO TO 970
  968 WRITE (6,972)
  972 FORMAT (//50X,'POINT VALUES',//)
  970 WRITE (6,971)
  971 FORMAT (10X,' NO ',5X,'LATITUDE',5X,'LONGITUDE',5X,'COMPUTED',9X,'
     1TRUE',7X,'DIFFERENCES', 6X,'PERCENTAGE',/47X,'VALUES',9X,'VALUES',
     210X,'C-T',7X,'OF NECESSARY MEAS.',/)
      I = 1
      JL = 2
      JLL = KK(ICKE)
  304 WRITE (6,68) I,B(ICKE),RR(I,1),RR(I,2),RR(I,5),RR(I,6),RR(I,3)
   68 FORMAT (I13,2F13.2,6X,F7.2,9X,F7.2,7X,F7.2,F16.2)
      DO 66 I=JL,JLL
   66 WRITE (6,67) I ,RR(I,1),RR(I,2),RR(I,5),RR(I,6),RR(I,3)
   67 FORMAT (I13, F26.2,6X,F7.2,9X,F7.2,7X,F7.2,F16.2)
      ICKE = ICKE+1
      JL = JLL+2
      I = I+1
      JLL = JLL+KK(ICKE)
      IF (ICKE-NM) 304,304,305
  305 WRITE (6,309) SUM,KE,SUMS,SMI
  309 FORMAT (///2X,'SUM OF SQUARES:',F12.2,/2X,'STANDARD DEVIATION FROM
     1',I5,' MEASUREMENTS:',F10.2,' MGAL',/2X,'FORMAL STANDARD DEVIATION
     2 FROM COLLOCATION:',F9.2,' MGAL',//)
  301 STOP
      END


      SUBROUTINE VMEAN (VM)
C
C     DETERMINES QUANTITIES FOR THE MEAN VALUE COMPUTATION
```

```
C         VM     MEAN SIDE LENGTH OF BLOCK IN MINUTES OF ARC
C         RAN    ZONE OF INFLUENCE ON EACH SIDE OF INTERPOLATION PROFILE.
C         NNV    NUMBER OF POINTS FOR ONE MEAN VALUE AND ONE FLIGHT PROFILE
C         MMV    NUMBER OF POINTS ON EACH FLIGHT PROFILE USED TO SET UP THE
C                ESTIMATION OPERATOR. MMV MUST BE ODD AND GREATER THAN 1.
C         MVV    SEPARATION OF THE ABOVE POINTS IN UNITS OF VVM
C         MV     NUMBER OF INTERPOLATION PROFILES FOR MEAN VALUE OF SIZE VM
C         NV     NUMBER OF POINTS ON EACH INTERPOLATION PROFILE USED FOR THE
C                ESTIMATION
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FB(16),FC(16)
     1,TRANS(12),SR(5),CO,COO,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
C
      MMV = 3
      MVV = 5
      NV = 1
      RAN = 1.5*VM*COO
      SMI = 1.5*SMI*COO
      IF (RAN-SMI) 901,902,902
  901 RAN = SMI
  902 SMI = 1.
      IF (VM-40.) 910,910,911
  911 MV = 5
      ICK = -1
      GO TO 912
  910 IF (VM-5.) 914,913,913
  913 MV = 3
      ICK = 0
      GO TO 912
  914 MV = 1
      ICK = 1
      IF (VM) 912,915,912
  915 VM = VVM
      NNV = 1
  912 RETURN
      END



      SUBROUTINE ZINF (ICKE,VM)
C
C     DETERMINES THE PROFILES TO BE USED FOR THE INTERPOLATION OF A SPECIFIC
C     POINT USING THE LATITUDE BLAT OF THIS POINT AND THE QUANTITY RAN AS FIXED
C     BY THE SUBROUTINE VMEAN.THE PROFILE NUMBERS ARE STORED IN THE ARRAY TRANS.
C     ICKE       NUMBER OF INTERPOLATION PROFILE
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FB(16),FC(16)
     1,TRANS(12),SR(5),CO,COO,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
C
C     SELECTION OF FLIGHT PROFILES FOR INTERPOLATION PROFILE NUMBERED ICKE
C
  416 NJ = 0
      BLAT = B(ICKE)
      BLO = C(ICKE)
      HEI = D(ICKE)
```

```
      NNV = VM*DCOS(BLAT)/VVM
      IF (MVV-5) 419,419,420
  419 MVV = 5
  420 IF (NNV-1) 421,421,422
  421 NNV = KK(ICKE)
  422 ITZ = 1
      STI = MVV
      STT = STI*VVM
      STI = MMV/2
      DO 410 I=1,10
  410 TRANS(I) = 0.
      BLAT1 = BLAT+RAN
      BLAT2 = BLAT-RAN
      I = 1
      J = 1
  401 BL= B(I)
      IF (BL.GT.BLAT1) GO TO 400
      IF (BL.LT.BLAT2) GO TO 400
      TRANS(J) = I
      BP(J) = B(I)
      CP(J) = BLD-STI*STT/DCOS(B(I))
      DP(J) = D(I)
      KKP(J) = MMV
      KKKP(J) = KKK(I)
      FB(J) = 0.
      FC(J) = STT/DCOS(B(I))
      DO 409 K=1,7
  409 IDP(J,K) = ID(I,K)
      NJ = J
      J = J+1
  400 I = I+1
      IF (I-NN) 401,401,402
  402 IF (NJ) 413,413,412
  413 WRITE (6,414) BLAT
  414 FORMAT (///2X,'NO FLIGHT PROFILES IN THE DEFINED REGION.INTERPOLAT
     1ION PROFILE WITH THE LATITUDE BLAT=',F7.2,'  CANNOT BE DETERMINED'
     2,///)
      IMM = IMM+KKP(ICKE)
          ICKE = ICKE+1
      IF (ICKE-NM) 416,415,417
  417 ITZ = -1
      GO TO 418
C
C     PATTERN OF INTERPOLATION POINTS FOR SPECIFIC MEAN VALUE
C     VALUES USED FOR INTERPOLATION OF ONE PROFILE ARE STORED IN /BSD/
C
  412 STI = MV
      ST = VM/STI
      J = NJ+1
      IF (ICK) 403,404,405
  403 BP(J) = BLAT+2.*ST
      BP(J+1) = BLAT+ST
      BP(J+2) = BLAT
      BP(J+3) = BLAT-ST
      BP(J+4) = BLAT-2.*ST
      GO TO 406
  404 BP(J) = BLAT+ST
      BP(J+1) = BLAT
      BP(J+2) = BLAT-ST
      GO TO 406
  405 BP(J) = BLAT
  406 DO 407 K=1,MV
```

```
      J = NJ+K
      CP(J) = BLO
      DP(J) = HEI
      KKP(J) = NV
      FB(J) = 0.
      FC(J) = STT/DCOS(BLAT)
      KKKP(J) = 1
      IDP(J,1) = 3
      DO 407 IK=2,7
  407 IDP(J,IK) = 0
  418 RETURN
      END



      SUBROUTINE PLAY (AA,BB,MOLO,MA,MB)
C
C     COMPUTES COVARIANCE MATRICES
C     MOLO      -1  AUTOCOVARIANCE MATRIX OF OBSERVATIONS CXX STORED IN
C                   ARRAY AA
C                0  CROSSCOVARIANCE MATRIX OF SIGNAL AND OBSERVATION CSX
C                   STORED IN ARRAY BB
C                1  AUTOCOVARIANCE MATRIX OF SIGNA CSS STORED IN ARRAY AA
C     MA,MB     DIMENSION PARAMETERS OF AA AND BB AS DEFINED IN CALLING PROGRAM
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FB(16),FC(16)
     1,TRANS(12),SR(5),CO,COO,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
      COMMON /CMCOV/CI(12),CR(51),SIGMAO(300),SIGMA(300),KI(25),NI,LOCAL
      DIMENSION AA(MA,MA),BB(MB,MA)
      DATA GM,RE/3.98014,6371.003/
C
C     INPUT POINT P
C
      JV =  NJ+MV
      IF (MOLO) 220,221,221
  220 I = 1
      GO TO 222
  221 I = NJ+1
  222 ILL = 0
      IJ = 0
  214 IJ = IJ+ILL
      ILL = 0
      DO 228 J=1,7
      IF (IDP(I,J)) 229,228,229
  229 ILL = ILL+1
  228 CONTINUE
      RB = BP(I)
      RC = CP(I)
      CR(10) = GM/(RE+DP(I))**2
      RFB = FB(I)
      RFC = FC(I)
      CR(2) = DP(I)
      J = 0
      KT = 0
  212 IJ = IJ+KT*ILL
      TJ = J
      ITT = 0
      RBR = RB+TJ*RFB
```

```
      RCR = RC+TJ*RFC
      CR(4) = DSIN (RBR)
      CR(6) = DCOS (RBR)
      IF (MOLO) 250,251,250
C
C     INPUT POINT 2
C
  251 II = 1
      IIJJ = 0
      GO TO 210
  250 II = I
      IIJJ = IJ
  210 RBB = BP(II)
      RCC = CP(II)
      CR(11) = GM/(RE+DP(II))**2
      RFBB = FB(II)
      RFCC = FC(II)
      CR(3) = DP(II)
      IF (I-II) 223,224,223
  223 JJ = 0
      GO TO 208
  224 JJ = J
  208 TJJ = JJ
      IIJJ = IIJJ+ITT*JLL
      JLL = 0
      DO 230 JK=1,7
      IF (IDP(II,JK)) 231,230,231
  231 JLL = JLL+1
C
C     COMPUTATION OF SPHERICAL DISTANCE
C
  230 CONTINUE
      RBBR = RBB+TJJ*RFBB
      RCCR = RCC+TJJ*RFCC
      SS = RCCR-RCR
      CR(5) = DSIN (RBBR)
      CR(7) = DCOS (RBBR)
      CR(8) = DSIN (SS)
      CR(9) = DCOS (SS)
      CR(1) = CR(4)*CR(5)+CR(6)*CR(7)*CR(9)
C
C     COVARIANCES
C
      IK = 1
  304 KI(6) = IDP(I,IK)
      IF (KI(6)) 300,310,302
  300 WRITE (6,303) I,KI(6)
  303 FORMAT (/20X,'ERROR IN INPUT SPECIFICATION FOR PROFILE',I3,',',I2)
  302 III = IJ+IK
      IF (I-II) 319,320,319
  320 IF (J-JJ) 319,321,319
  321 IKK = IK
      GO TO 309
  319 IKK = 1
  309 KI(7) = IDP(II,IKK)
      IF (KI(7)) 306,307,308
  306 WRITE (6,303) II,KI(7)
  308 JJJ = IIJJ+IKK
      CALL COVBX
      CALL COVCX (COV)
      IF (DABS(COV)-.1) 633,633,634
  633 COV = 0.
```

```
  634 IF (MOLO) 316,317,316
  316 AA(III,JJJ) = COV
      GO TO 307
  317 BB(III,JJJ) = COV
C
C     SETUP OF COMPLETE MATRIX
C
  307 IKK = IKK+1
      IF (IKK-7) 309,309,310
  310 IK = IK+1
      IF (IK-7) 304,304,305
  305 JJ = JJ+1
      ITT = 1
      IF (JJ-KKP(II)) 208,209,209
  209 II = II+1
      IF (MOLO) 252,252,253
  253 IF (II-JV) 210,210,254
  254 GO TO 211
  252 IF (II-NJ) 210,210,211
  211 J = J+1
      KT = 1
      IF (J-KKP(I)) 212,213,213
  213 I = I+1
      IF (MOLO) 225,226,226
  225 IF (I-NJ) 214,214,215
  215 GO TO 227
  226 IF (I-JV) 214,214,227
  227 RETURN
      END


      SUBROUTINE ORDER (AA,MA,M,NDEC)
C
C     SHIFTS THE UPPER TRIANGULAR PART OF THE REAL SYMMETRIC MATRIX AA
C     INTO THE FIRST MA*(MA-1)/2 STORAGE LOCATIONS OF THIS ARRAY AND
C     VICE VERSA
C     MA         DIMENSION OF AA AS DEFINED IN CALLING PROGRAM
C     M          ACTUAL DIMENSION OF MATRIX
C     NDEC       -1  TWO-DIMENSIONAL ARRAY INTO ONE-DIMENSIONAL ARRAY
C                 1  OINEDIMENSIONAL ARRAY INTO TWO-DIMENSIONAL ARRAY
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      DIMENSION AA(MA,MA)
C
      IF (NDEC) 300,300,301
C
C     TWO-DIMENSIONAL ARRAY INTO ONE-DIMENSIONAL ARRAY
C
  300 I = 1
      J = 1
      K = 1
      N = 1
  304 AA(I,J) = AA(K,N)
      K = K+1
      IF (K-N) 302,302,303
  302 I = I+1
      IF (I-MA) 304,304,305
  305 J = J+1
      I = 1
      GO TO 304
```

```
  303 N = N+1
      K = 1
      IF (N-M) 302,302,307
  307 GO TO 308
C
C     ONE-DIMENSIONAL ARRAY INTO TWO-DIMENSIONAL ARRAY
C
  301 MM = M*(M+1)/2
      MC = MM/MA
      MR = MM-MC*MA
      I = M
      J = M
      K = MR
      N = MC+1
      IF (K) 309,310,311
  309 WRITE (6,312) N,K
  312 FORMAT (10X,'ERROR IN POSITION',2I3)
  310 K = MA
      N = N-1
  311 AA(I,J) = AA(K,N)
      K = K-1
      IF (K) 313,313,314
  314 I = I-1
      IF (I) 315,315,311
  315 J = J-1
      I = J
      GO TO 311
  313 N = N-1
      K = MA
      IF (N) 308,308,314
  308 RETURN
      END



      SUBROUTINE DATS (KKL,GN,GD,GDD,IZI)
C
C     READS THE SIMULATED 'ERRORLESS' DATA FROM A DIRECT ACCESS FILE AND
C     CORRUPTS THESE VALUES BY DIFFERENT KINDS OF ERRORS. IN THE PRESENT
C     SETUP SIX MEASUREMENTS ARE READ FOR EACH POINT, THE FIRST THREE
C     OF WHICH ARE USED.
C     KKL     NUMBER OF RECORDS TO BE READ FROM EACH FLIGHT FILE
C     IZI        -1  NORMAL DEVIATES WITH VARIANCES GN,GD,GDD ARE ADDED TO
C                    TRUE VALUES
C                 0  YULE TIME SERIES WITH CORRELATIONS 0.733 AND 0.307 IS
C                    ADDED TO GRAVITY ANOMALIES
C                 1  LINEAR SYSTEMATIC ERROR IS ADDED TO GRAVITY ANOMALIES
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FR(16),FC(16)
     1,TRANS(12),SR(5),CD,CDD,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
      COMMON /SHT/ ZZ(3,6)
      DIMENSION GN(10),GD(10),GDD(10)
C
C     READ DATA
C
      DEFINE FILE 11(7,2400,L,K11)
      DEFINE FILE 12(7,2400,L,K12)
      DEFINE FILE 13(7,2400,L,K13)
```

```
      DEFINE FILE 14(7,2400,L,K14)
      DEFINE FILE 15(7,2400,L,K15)
      DEFINE FILE 16(7,2400,L,K16)
      DEFINE FILE 17(7,2400,L,K17)
      DEFINE FILE 18(7,2400,L,K18)
      DEFINE FILE 19(7,2400,L,K19)
      DEFINE FILE 20(7,2400,L,K20)
      DEFINE FILE 21(7,2400,L,K21)
      IZ = 591
      IKK = KKL*600
      AM = 0.
      I = 1
      IX = KKK(I)
  502 J1 = 1
      JK = TRANS(I)+10.
      JJ = 1
  500 J2 = J1+599
      READ (JK' JJ) (Z(I,J),J=J1,J2)
      J1 = J1+600
      JJ = JJ+1
      IF (JJ-KKL) 500,500,501
  501 I = I+1
      IF (I-NJ) 502,502,503
  503 IF (IZI) 507,508,509
C
C     NORMAL DEVIATES
C
  507 DO 506 I=1,NJ
      IK = 6*KK(I)
      K = -1
      JK = TRANS(I)
      DO 505 J=1,IK,6
      K = K+1
      S = GD(JK)
      CALL GAUSS (IZ,S,AM,V)
      ZZ(I,1) = Z(I,J)+V
      S = GDD(JK)
      CALL GAUSS (IZ,S,AM,V)
      ZZ(I,2) = Z(I,J+1)+V
      CALL GAUSS (IZ,S,AM,V)
      ZZ(I,3) = Z(I,J+2)+V
      DO 505 JJ=1,IX
      II = K*IX+JJ
  505 Z(I,II) = ZZ(I,JJ)
  506 CONTINUE
      GO TO 510
C
C     CORRELATED ERRORS
C
  508 DO 511 I=1,NJ
      IK = 6*KK(I)
      K = -1
      JK = TRANS (I)
      S = GD(JK)
      CALL GAUSS (IZ,S,AM,U)
      CALL GAUSS (IZ,S,AM,UU)
      V = -1.1
      VV = .5
      DO 512 J=1,IK,6
      K = K+1
      CALL YULE (U,UU,V,VV,S,SS,IZ)
      ZZ(I,1) = Z(I,J)+SS
```

```
      ZZ(I,2) = Z(I,J+1)
      ZZ(I,3) = Z(I,J+2)
      DO 512 JJ=1,IX
      II = K*IX+JJ
  512 Z(I,II) = ZZ(I,JJ)
  511 CONTINUE
C
C     SYSTEMATIC ERRORS
C
      GO TO 510
  509 DO 513 I=1,NJ
      IK = 6*KK(I)
      K = -1
      JK = TRANS (I)
      S = 5.
      SS = KKL*100
      S = S/SS
      SS = 0.
      DO 514 J=1,IK,6
      K = K+1
      SS = SS+S
      ZZ(I,1) = Z(I,J)+SS
      ZZ(I,2) = Z(I,J+1)
      ZZ(I,3) = Z(I,J+2)
      DO 514 JJ=1,IX
      II = K*IX+JJ
  514 Z(I,II) = ZZ(I,JJ)
  513 CONTINUE
  510 DO 515 I=1,NJ
      IK = KK(I)*KKK(I)+1
      DO 516 J=IK,IKK
  516 Z(I,J) = 99999.
  515 CONTINUE
      RETURN
      END



      SUBROUTINE SCAN (AA,MA,IMM,VM,RR,ICKE)
C
C     COMPUTES ONE PROFILE OF POINT OR MEAN VALUES
C     AA        PREDICTION MATRIX
C     IMM       SEQUENTIAL NUMBER OF MEAN OR POINT VALUE
C     RR        ARRAY TO STORE THE COMPUTED VALUES AND THEIR COORDINATES
C     ALL OTHER QUANTITIES ARE AS DEFINED IN CALLING PROGRAM
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FP(16),FC(16)
     1,TRANS(12),GR(5),CO,COD,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITI,IFI
      DIMENSION AA(MA,MA),RR(612,6)
C
C     COMPARE INITIAL POINTS OF FLIGHT AND INTERPOLATION PROFILES
C
      JJKK = (KK(1)-1)*KKK(1)-2*MMV*MVV
      IF (IDI) 622,622,623
  623 SVM = VM
      GO TO 624
  622 SVM = 0.
  624 SUM = MVV*(MMV/2)
```

```
      BLO = C(ICKE)-SVM
      BLAT = B(ICKE)
      BL = BLO-SUM*VVM/(DCOS(BLAT))-SVM/2.
      KM = KK(ICKE)
      DO 616 I=1,NJ
  616 INI(I) = 0
      IM = 1
  607 SUM = IM-1
      SUMI = 0.
      BLO = BLO+SVM
      KR = NNV
      SUMS = KR
      IR = 1
      BL = BL+SVM
      NC = TRANS(1)
      CPP = C(NC)
      CDD = (BL-CPP)*DCOS(BLAT)/VVM
      IF (CDD) 600,601,602
  600 IF (CDD+.5) 603,601,601
  603 NC = CDD-.5
      KRR = KR+NC
      IF (KRR) 604,604,605
  604 IC = ICKE-NN
      WRITE (6,606) IC,IM,IMM
  606 FORMAT (//2X,'INTERPOLATION PROFILE NO',I4,'    POINT NO',I4,'
     1SEQUENTIAL NO',I4,/2X,  'HAS NOT BEEN COMPUTED BECAUSE NO MEASUREM
     2ENTS ARE AVAILABLE',/)
      RR(IMM,1) = BLO
      RR(IMM,2) = 999999999.
      RR(IMM,3) = 0.
      IM = IM+1
      IMM = IMM+1
      IF (IM-KM) 607,607,630
  630 GO TO 615
  605 SUM = -NC
      CDD = KR
      CN = 100.-SUM*100./CDD
      RR(IMM,3) = CN
      SUMS = KR+NC
      IR = IR-NC
      IC = ICKE-NN
      WRITE (6,608) IC,IM,IMM,CN
  608 FORMAT (//2X,'INTERPOLATION PROFILE NO',I4,'    POINT NO',I4,'
     1SEQUENTIAL NO',I4,/2X,  'HAS BEEN COMPUTED WITH ONLY',F5.1,'  PERC
     2ENT OF THE MEASUREMENTS REQUIRED',/)
      GO TO 601
  602 IF (CDD-.5) 601,601,609
  609 DO 610 I=1,NJ
      CDD = (BL-CPP)*DCOS(BP(I))/VVM
      NC = CDD+.5
  610 INI(I) = NC*KKKP(I)
  601 IF (IDI) 625,625,626
C
C     MEAN VALUES
C
  626 CALL FIX (AA,MA,SUM,JJ)
      SUMI = SUMI+SUM
      IF (INI(1)-JJKK) 617,613,618
  618 SUMS = IR
      CDD = KR
      IF (IR-KR) 620,621,620
  620 CN = SUMS*100./CDD
```

```
      RR(IMM,3) = CN
      IC = ICKE-NN
      WRITE (6,608) IC,IM,IMM,CN
  621 IM = KM
      GO TO 619
  617 IR = IR+1
      DO 612 I=1,NJ
  612 INI(I) = INI(I)+KKKP(I)
      IF (IR-KR) 626,625,619
  619 SUMI = SUMI/SUMS
      RR(IMM,1) = BLO
      RR(IMM,2) = SUMI
      IMM = IMM+1
      IM = IM+1
      IF (IM-KM) 607,607,629
  629 GO TO 615
C
C     POINT VALUES
C
  625 CALL FIX (AA,MA,SUM,JJ)
      RR(IMM,1) = BLO
      RR(IMM,2) = SUM
      BLO = BLO+VM/DCOS(BLAT)
      IMM = IMM+1
      IM = IM+1
      DO 627 I=1,NJ
  627 INI(I) = INI(I)+KKKP(I)
      IF (IM-KM) 628,628,615
  628 IF (INI(1)+KKKP(1)-JJKK) 625,631,631
  631 DO 632 I=IMM,KM
  632 RR(I,2) = 999999999.
  615 RETURN
      END


      SUBROUTINE FIX (AA,MA,SUM,JJ)
C
C     PERFORMS ONE OPERATION OF THE PREDICTION MATRIX AA ON THE OBSERVATION
C     FIELD Z. RESULT IS STORED IN SUM.
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FB(16),FC(16)
     1,TRANS(12),SR(5),CD,COD,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IOP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
      DIMENSION AA(MA,MA)
      I = 1
  703 SUM = 0.
      J = 0
      II = 1
      JLL = 0
  701 ILL = 0
      KKKK = KKKP(II)
      KO = MVV*KKKK
      ILL = ILL-KO+INI(II)
      JLL = JLL-KKKK
      DO 700 IL = 1,MMV
      ILL = ILL+KO
      JLL = JLL+KKKK
      DO 700 IZ=1,KKKK
```

```
      J = JLL+IZ
      JJ = ILL+IZ
  700 SUM = SUM+AA(I,J)*Z(II,JJ)
      JLL = JLL+KKKK
      II = II+1
      IF (II-NJ) 701,701,702
  702 SR(I) = SUM
      I = I+1
      IF (I-MV) 703,703,704
  704 SUM = 0.
      DO 706 I=1,MV
  706 SUM = SUM+SR(I)
      SUMI = MV
      SUM = SUM/SUMI
      RETURN
      END



      SUBROUTINE COMPA (RR,KE,SUM,SUMS)
C
C     DETERMINES THE DIFFERENCES BETWEEN TRUE AND ESTIMATED VALUES OF THE
C     SIMULATED MODEL. THE SUM OF SQUARES AND THE STANDARD DEVIATION ARE
C     STORED IN SUM AND SUMS
C     RR        ARRAY CONTAINING THE TRUE AND THE ESTIMATED VALUES
C     KE        TOTAL NUMBER OF ESTIMATED VALUES
C     ITZ       TOTAL NUMBER OF TRUE VALUES
C     SUM       SUM OF SQUARES OF DIFFERENCES
C     SUMS      VARIANCE OF DIFFERENCES
C
      IMPLICIT REAL *8(A-H,O-Y),LOGICAL(L)
      COMMON /TICK/ BP(12),CP(12),DP(12),B(16),C(16),D(16),FB(16),FC(16)
     1,TRANS(12),SR(5),CO,COO,VVM,SMI,RAN,Z(5,4200),KKP(12),KKKP(12),IDP
     2(12,7),KK(16),KKK(16),ID(16,7),NN,MM,NM,NNN,INI(12),NNV,NV,MV,ICK,
     3NJ,MVV,MMV,IDI,ITZ,IFI
      DIMENSION RR(612,6)
      SUM = 0.
      KKKK = 0
      IDD = 0
      ILL = 1
      J = 1
      JJ = 1
      NNNN = NNN
  706 BLAT = B(NNNN)
  702 IF (DABS(BLAT-RR(J,4))-.01) 701,701,700
  700 J = J+1
      IF (J-IFI) 702,702,703
  703 IF (ILL) 714,714,713
  713 WRITE (6,710)
  710 FORMAT (///2X,'TRUE AND ESTIMATED VALUES DO NOT HAVE COMPATIBLE CO
     1ORDINATES',///)
      GO TO 709
  701 BLO = RR(JJ,1)
  711 IF (DABS(BLO-RR(J,5))-.01) 704,704,712
  704 KKKN = KK(NNNN)
      KKKK = KKKK+KKKN
      DO 705 I=JJ,KKKK
      II = J-JJ+I
      RR(I,6) = RR(II,6)
  705 RR(I,5) = RR(II,5)
      ILL = -1
```

```
      JJ = JJ+KKKN
      J = J+KKKN
      NNNN = NNNN+1
      IF (NNNN-NM) 706,706,707
  707 IF (ILL) 714,714,713
  714 DO 708 I=1,KE
      RR(I,4) = RR(I,5)
      RR(I,5) = RR(I,6)
      IF (RR(I,2) -999999999.) 716,717,716
  716 SS = RR(I,2)-RR(I,5)
      SUM = SUM+SS*SS
      RR(I,5) = SS
      GO TO 708
  717 IDD = IDD+1
      RR(I,6) = 999999999.
      RR(I,3) = 0.
  708 CONTINUE
      SS = KE-IDD
      KE = SS
      SUMS  = DSQRT (SUM/SS)
      GO TO 709
  712 J = J+1
      IF (J-IFI) 711,711,715
  715 IF (ILL) 709,709,713
  709 RETURN
      END


      SUBROUTINE YULE (U,UU,V,VV,S,SS,IZ)
C
C     COMPUTES A STATIONARY TIME SERIES USING YULE'S SCHEME
C     U,UU      COEFFICIENTS OF THE SERIES FOR THE TWO PRECEDING EVENTS
C     V,VV      THE TWO PRECEDING EVENTS OF THE SERIES
C     S         VARIANCE OF THE GAUSSIAN PROCESS USED TO GENERATE THE SERIES
C     IZ        RANDOM NUMBER TO START GAUSSIAN PROCESS
C     SS        TRANSFER OF COMPUTED EVENT
C
      AM = 0.
      CALL GAUSS (IZ,S,AM,E)
      SS = E-U*V-UU*VV
      UU = U
      U = SS
      RETURN
      END


C     SUBROUTINE COVAX
C
C    A LISTING OF THIS SUBROUTINE CAN BE FOUND IN TSCHERNING (1975)
C     OSU REPORT NO 225. THE FOLLOWING CORRECTIONS SHOULD BE MADE
C     (PERSONAL COMMUNICATION BY C.C.TSCHERNING) :
C
C     PAGE 59,LINE 23  **RL1)*RL2+(2.+(5.+(6.+D2*RL1)*RL1)*RL1)*RN)*RN)*RN)**NL
C     PAGE 60,STATEMENT 31  CHANGE SIGN
C     PAGE 60,STATEMENT 33  CHANGE SIGN
C
C     THE PAGE NUMBERS REFER TO THE ABOVE REPORT
C
```

Appendix B: Sample Computations

S A M P L E I N P U T

```
.99400000+00  .97104000+01    1    0    0   12 T
  7       498.6         10.       7
-17.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
-18.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
-19.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
-20.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
-21.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
-22.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
-23.     60.     10.      0.      1.      1.     612  3  3  5 10  0  0  0  0
  1    2   10  150        60.         60.
-17.5   60.5    0.    15
-18.5   60.5    0.    10
```

DATA CONTROL

```
PARAMETERS SPECIFYING THE DEGREE-VARIANCE MODEL

          S =        0.994000
          A =         9.7104
          K1(5) =     1
          K2 =        0
          K3 =        0
          N =        12
          LOCAL =     1

MEAN VELOCITY OF AIRCRAFT IN KNOTS PER HOUR:    498.60
MEAN INTERVAL BETWEEN MEASURING POINTS IN SECONDS OF TIME:  10.00
SIZE OF MEAN ANOMALY BLOCK:  60.* 60, MINUTES
MEAN PROFILE SPACING:  60.00 MINUTES

FLIGHT PROFILE NO  1
INITIAL POINT OF PROFILE   LATITUDE: -17.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

FLIGHT PROFILE NO  2
INITIAL POINT OF PROFILE   LATITUDE: -18.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

FLIGHT PROFILE NO  3
INITIAL POINT OF PROFILE   LATITUDE: -19.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

FLIGHT PROFILE NO  4
INITIAL POINT OF PROFILE   LATITUDE: -20.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

FLIGHT PROFILE NO  5
INITIAL POINT OF PROFILE   LATITUDE: -21.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

FLIGHT PROFILE NO  6
INITIAL POINT OF PROFILE   LATITUDE: -22.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

FLIGHT PROFILE NO  7
INITIAL POINT OF PROFILE   LATITUDE: -23.00   LONGITUDE:  60.00  ALTITUDE: 10000.
NUMBER OF POINTS IN PROFILE: 612
VARIANCE  OF GEOID UNDULATIONS:   0.0    OF DELTA G:   1.00   OF SECOND ORDER DERIVATIVES:   1.00
SPECIFICATION NUMBERS:  3  5 10  0  0  0  0

INTERPOLATION PROFILE NO  1
INITIAL POINT OF PROFILE   LATITUDE: -17.50   LONGITUDE:  60.50  ALTITUDE:      0.
NUMBER OF POINTS IN PROFILE:  15
INTERPOLATION PROFILE NO  2
INITIAL POINT OF PROFILE   LATITUDE: -18.50   LONGITUDE:  60.50  ALTITUDE:      0.
NUMBER OF POINTS IN PROFILE:  10
```

C O M P U T A T I O N S

INTERPOLATION PROFILE NO 1 USES FLIGHT PROFILES NOS 1. 2.

INVERSION OF COVARIANCE MATRIX OKAY

INTERPOLATION PROFILE NO 1 POINT NO 1 SEQUENTIAL NO 1
HAS BEEN COMPUTED WITH ONLY 87.8 PERCENT OF THE MEASUREMENTS REQUIRED

INTERPOLATION PROFILE NO 1 POINT NO 15 SEQUENTIAL NO 15
HAS BEEN COMPUTED WITH ONLY 65.9 PERCENT OF THE MEASUREMENTS REQUIRED

INTERPOLATION PROFILE NO 2 USES FLIGHT PROFILES NOS 1. 2. 3.

INVERSION OF COVARIANCE MATRIX OKAY

INTERPOLATION PROFILE NO 2 POINT NO 1 SEQUENTIAL NO 16
HAS BEEN COMPUTED WITH ONLY 87.8 PERCENT OF THE MEASUREMENTS REQUIRED

MEAN VALUES FOR BLOCKS OF 60. * 60. MINUTES

NO	LATITUDE	LONGITUDE	COMPUTED VALUES	TRUE VALUES	DIFFERENCES C-T	PERCENTAGE OF NECESSARY MEAS.
1	-17.50	60.50	-24.26	-22.35	-1.91	87.80
2		61.50	-43.19	-41.26	-1.93	100.00
3		62.50	-14.31	-11.92	-2.39	100.00
4		63.50	10.52	8.35	2.17	100.00
5		64.50	20.84	20.58	0.26	100.00
6		65.50	-1.16	-6.91	5.75	100.00
7		66.50	12.45	20.05	-7.60	100.00
8		67.50	12.59	13.52	-0.93	100.00
9		68.50	-47.19	-45.19	-2.00	100.00
10		69.50	4.82	1.96	2.86	100.00
11		70.50	59.70	57.72	1.98	100.00
12		71.50	13.24	16.23	-2.99	100.00
13		72.50	-0.59	-1.09	0.50	100.00
14		73.50	-7.55	-10.29	2.74	100.00
15		74.50	3.13	10.89	-7.76	65.88
16	-18.50	60.50	31.36	24.09	7.27	87.80
17		61.50	8.16	13.46	-5.30	100.00
18		62.50	-8.87	-8.78	-0.09	100.00
19		63.50	26.39	21.63	4.76	100.00
20		64.50	44.88	46.76	-1.88	100.00
21		65.50	13.46	6.09	7.37	100.00
22		66.50	23.50	25.67	-2.17	100.00
23		67.50	-5.99	-13.27	7.28	100.00
24		68.50	43.74	42.01	1.73	100.00
25		69.50	21.97	13.61	8.36	100.00

SUM OF SQUARES: 494.63
STANDARD DEVIATION FROM 25 MEASUREMENTS: 4.45 MGAL
FORMAL STANDARD DEVIATION FROM COLLOCATION: 6.05 MGAL